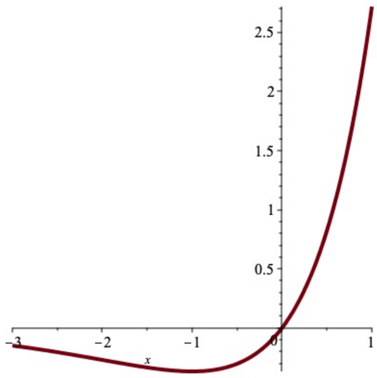
MATH 122: Calculus II

Some Notes on Assignment 9

I: Section 6.3: 37, 47, 54

Exercise 37 : For $f(x) = xe^x$, the product rule gives $f'(x) = 1e^x + xe^x = (1+x)e^x$ and $f''(x) = 1e^x + (1+x)e^x = (2+x)e^x$. Since e^x is positive for all x, the sign of the first derivative is the sign of $1+x$ and the sign of the second derivative is the sign of $2+x$. Thus f' is negative for $x < -1$ and positive for $x > -1$. There is a relative minimum at $x = -1$ where the value of the function f is $-1e^{-1} = \frac{-1}{e}$. The second derivative f'' is negative for $x < -2$ and positive for $x > -2$. There is a point of inflection on the graph of f at $(-2, f(-2)) = (-2, \frac{-2}{e^2})$ where the graph changes concavity from negative to positive.



Exercise 37: Graph of xe^x

Exercise 47: $h(x) = 79.041 + 6.39x - e^{3.261-0.993x}$ has $h'(x) = 6.39 + 0.993xe^{3.261-0.993x}$ and $h''(x) = -.986049e^{3.261-0.993x}$.
(a) Height at age 1 $= h(1) = 75.771\ cm$ and rate of growth $= h'(1) = 15.982\ cm/yr$.
(b) $h''(x)$ is negative for all x so rate of growth is always decreasing. It is largest at $x = 1/4$ year = 3 months and smallest at $x = 6$ years.

Exercise 54: Revenue $R(x) =$ (demand)(price) $= xp_0e^{-ax}$ so $R'(x) = p_0e^{-ax}(1-ax)$ which switches from positive to negative at $x = \frac{1}{a}$. We maximize revenue when $x = \frac{1}{a}$ where $p = p_0e^{-1} = \frac{p_0}{e}$. For $p_0 = 300$, this is about \$110.36. The maximum revenue would be \$15000$e^{-1} \approx$ \$5518.19

II: Section 6.4: 19, 25, 31

Exercise 19: Let $u = 1 + 2\cos x$ so $du = -2\sin x\, dx$ and $\frac{-1}{2}du = \sin x\, dx$. Then

$$\int \frac{3\sin x}{1+2\cos x}\, dx = \int \frac{-3}{2}\frac{1}{u}\, du = \frac{-3}{2}\ln|u| + C = \frac{-3}{2}\ln|1+2\cos x| + C$$

Exercise 25: Write the integral as $\int \frac{\cot x^{1/3}}{x^{2/3}}\, dx = \int (\cot x^{1/3})x^{-2/3}\, dx$. Note that $\left(x^{1/3}\right)' = \frac{1}{3}x^{-2/3}$ and $\int \cot u\, du = \int \frac{\cos u}{\sin u}\, du = \ln|\sin u| + C$. Let $u = x^{1/3}$ so $3\ du = x^{-2/3}dx$. Then $\int (\cot x^{1/3})x^{-2/3}\, dx = 3\int \cot u\, du = 3\ln|\sin u| + C = 3\ln|\sin x^{1/3}| + C$

Exercise 31: $\int \frac{\cos^2 x}{\sin x} dx = \int \frac{1-\sin^2}{\sin x} dx = \int \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} dx = \int \csc x - \sin x dx = \ln|\csc x - \cot x| + \cos x + C$

III: Section 6.5: 1, 8, 17

Exercise 1: Let $f(x) = 7^x$. Then $\ln f(x) = \ln 7^x = x\ln 7$ so $\frac{f'(x)}{f(x)} = \ln 7$ and hence $f'(x) = (\ln 7)f(x) = (\ln 7)7^x$.

Exercise 8: Let $f(x) = 3^{2-x^2}$ so $\ln f(x) = (2-x^2)\ln 3$ and $\frac{f'(x)}{f(x)} = (-2\ln 3)x$ and $f'(x) = (-2\ln 3)x3^{2-x^2}$

Exercise 17: Since e is a constant, we can use the Power Rule on the first term. The derivative of $x^e + e^x$ is $ex^{e-1} + e^x$.