MATH 122: Calculus II

Some Notes on Assignment 8

I: Section 6.2: 44, 51

Exercise 44 : $y = \frac{(x^2+3)^{2/3}(3x-4)^4}{\sqrt{x}} = \frac{(x^2+3)^{2/3}(3x-4)^4}{x^{1/2}}$. Using the properties that $\ln \frac{AB}{C} = \ln A + \ln B - \ln C$ and $\ln A^n = n \ln A$, we have $\ln y = \frac{2}{3}\ln(x^2+3) + 4\ln(3x-4) - \frac{1}{2}\ln x$. Differentiating: $y'\frac{1}{y} = \frac{4}{3}x\frac{1}{x^2+3} + 12\frac{1}{3x-4} - \frac{1}{2}\frac{1}{x}$ so $y' = y\left[\frac{4x}{3(x^2+3)} + \frac{12}{3x-4} - \frac{1}{2x}\right] = \frac{(x^2+3)^{2/3}(3x-4)^4}{\sqrt{x}}\left[\frac{4x}{3(x^2+3)} + \frac{12}{3x-}\right.$

Exercise 51: $s(t) = ct + \frac{c}{b}(m_1 + m_2 - bt)\left(\ln \frac{m_1+m_2-bt}{m_1+m2}\right)$. Well need to use the product rule to find the derivative of the second term: velocity $= s'(t) = c + \frac{c}{b}(-b)\left(\ln \frac{m_1+m_2-bt}{m_1+m2}\right) + \frac{c}{b}(m_1+m_2-bt)(-b)\frac{1}{m_1+m_2}\frac{m_1+m_2}{m_1+m_2-bt}$ which simplifies to $s'(t) = c - c\left(\ln \frac{m_1+m_2-bt}{m_1+m2}\right) - c = -c\left(\ln \frac{m_1+m_2-bt}{m_1+m2}\right)$

and the acceleration $= s''(t) = (-c)\frac{-b}{m_1+m2}\frac{m_1+m_2}{m_1+m_2-bt} = \frac{bc}{m_1+m_2-bt}$

(a) Initial velocity $= s'(0) = -c\ln\frac{m_1+m_2-b0}{m_1+m_2} = -c\ln 1 = 0$ and initial acceleration $= s''(0) = \frac{bc}{m_1+m_2-b0} = \frac{bc}{m_1+m_2}$

(b) Burnout: $s'\left(\frac{m_2}{b}\right) = -c\ln\left(\frac{m_1+m_2-m_2}{m_1+m_2}\right) = -c\ln\left(\frac{m_1}{m_1+m_2}\right) = c\ln\left(\frac{m_1+m_2}{m_1}\right), s''\left(\frac{m_2}{b}\right) = \frac{bc}{m_1+m_2-m_2} = \frac{bc}{m_1}$

II: Section 6.3: 20, 25, 32

Exercise 20: $f(x) = \ln e^x = x$ so $f'(x) = 1$.

Exercise 25: Use Product Rule on $f(x) = e^{3x}\tan\sqrt{x}$ to obtain $f'(x) = 3e^{3x}\tan\sqrt{x} + e^{3x}(\sec^2\sqrt{x})(\frac{1}{2\sqrt{x}})$

Exercise 32: Implicit differentiation of $xe^y + 2x - \ln(y+1) = 3$ first yields $e^y + xe^y y - y'\frac{1}{1+y} = 0$. Collecting terms, we have $e^y + 2 = y'\left(\frac{1}{y+1} - xe^y\right)$ so

$$y' = \frac{e^y + 2}{\frac{1}{y+1} - xe^y}$$

III: Section 6.4: 1, 7, 14

Exercise 1: Let $u = 2x + 7$ so $du = 2dx$ and $dx = \frac{1}{2}du$. Then $\int \frac{1}{2x+7}\,dx = \int \frac{1}{u}\frac{1}{2}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(|2x+7|) + C$. Thus $\int_{-2}^{1} \frac{1}{2x+7}\,dx = \frac{1}{2}\ln(2+7) - \frac{1}{2}\ln(-4+7) = \frac{1}{2}(\ln 9 - \ln 3) = \frac{1}{2}\ln\frac{9}{3} = \frac{1}{2}\ln 3 = \ln\sqrt{3}$

Exercise 7: $\int \tan 2x\,dx = \int \frac{\sin 2x}{\cos 2x}\,dx$. Let $u = \cos 2x$ so $du = -2\sin 2x dx$. The integral becomes $\int -\frac{1}{2}\frac{1}{u}\,du = -\frac{1}{2}\ln|u| + C = -\frac{1}{2}\ln|\cos 2x| + C$.

Thus $\int_0^{\pi/8}\tan 2x\,dx = -\frac{1}{2}(\ln|\cos\pi/4| - \ln|\cos 0|) = -\frac{1}{2}\left(\ln\sqrt{2}/2 - \ln 1\right) = -\frac{1}{2}\ln\frac{\sqrt{2}}{2} = -\frac{1}{2}\ln(1/sqrt2) = -\frac{1}{2}\ln(2^{-1/2}) = (-\frac{1}{2})(-\frac{1}{2})\ln 2 = \frac{\ln 2}{4}$.

Exercise 14: Let $u = (2 + \ln x)$ so $du = \frac{1}{x}\,dx$. Then $\int(2+\ln x)^{10}\frac{1}{x}\,dx = \int u^{10}\,du = \frac{u^{11}}{11} + C = \frac{(2+\ln x)^{11}}{11} + C$