MATH 122: Calculus II

Some Notes on Assignment 7

I: Section 6.1: 27, 35

Exercise 27 : $f(x) = 4 - x^2$ has $f'(x) = -2x$ which is negative for all $x > 0$ so f has an inverse g. Since $y = 4 - x^2$, we have $x^2 = 4 - y$ so $x = \sqrt{4-y}$. The inverse function is $g(x) = \sqrt{4-x}$. Now domain of $g =$ range of $f = (-\infty, 4]$. Finally, $g'(x) = \frac{1}{f'(g(x))} = \frac{1}{-2g(x)} = \frac{1}{-2\sqrt{4-x}}$

Exercise 35: The derivative of $f(x) = x^3 + 4x - 1$ is $f'(x) = 3x^2 + 4$, a quantity that is always positive, guaranteeing that f has an inverse g. We also have $g(15) = 2$. Hence $g'(15) = \frac{1}{f'(g(15))} = \frac{1}{f'(2)} = \frac{1}{3(2^2)+4} = \frac{1}{16}$ is the slope of the tangent line to the graph of g at (15,2).

II: Section 6.2: 26, 33, 37

Exercise 26: If $g(x) = \ln(2x) = \ln 2 + \ln x$, then $g'(x) = 0 + \frac{1}{x} = \frac{1}{x}$. Thus $(\cos(\ln 2x))' = -\sin(\ln 2x) \times \frac{1}{x}$

Exercise 33: For $f(x) = \ln(\sec x + \tan x)$: $f(x) = \frac{(\sec x + \tan x)'}{\sec x + \tan x} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x} = \sec x$

Exercise 37: Use product rule on both terms on the left of $x \ln y - y \ln x = 1$. Implicit differentiation yields $1 \ln y + \frac{x}{y} y' - y' \ln x - \frac{y}{x} = 0$. Rearranging terms yields $\ln y - \frac{y}{x} = y' \left(\ln x - \frac{x}{y} \right)$, or equivalently $\frac{x \ln y - y}{x} = y' \left(\frac{y \ln x - x}{y} \right)$. Solve for $y' = \frac{y(x \ln y - y)}{x(y \ln x - x)}$.

III: Section 6.3: 1, 7, 14

Recall: If $f(x) = e^{g(x)}$, then $f'(x) = e^{g(x)} g'(x)$

Exercise 1: The derivative of e^{-5x} is $e^{-5x}(-5) = -5e^{-5x}$.

Exercise 7: The derivative of $e^{\sqrt{x+1}}$ is $e^{\sqrt{x+1}} \frac{1}{2\sqrt{x+1}}$

Exercise 14: The derivative of $(e^{3x} - e^{-3x})^4$ is $4(e^{3x} - e^{-3x})^3(3e^{3x} + 3e^{-3x})$ $= 12(e^{3x} - e^{-3x})^3(e^{3x} + e^{-3x})$