MATH 122: Calculus II

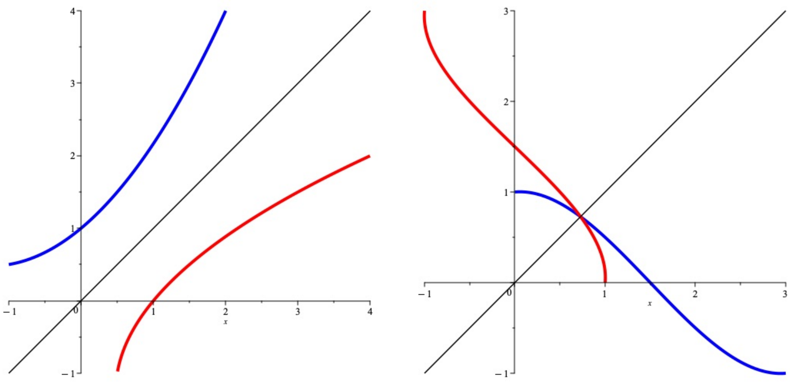
Some Notes on Assignment 6

I: Section 6.1: 14, 15, 18

Exercise 14 : (i): If $P(a, b)$ is on the graph of f, then $f(a) = b$ so $f^{-1}(b) = a$ which means $Q(b, a)$ is on the graph of f^{-1}
(ii) The midpoint of a line segment between (a_1, b_1) and (a_2, b_2) is $\left(\frac{a_1+a_2}{2}, \frac{b_1+b_2}{2}\right)$ so the midpoint of PQ is $\left(\frac{a+b}{2}, \frac{b+a}{2}\right) = \left(\frac{a+b}{2}, \frac{a+b}{2}\right)$ has equal coordinates and lies on the line $y = x$.
(ii) The line $y = x$ has slope 1 while the line segment PQ has slope $\frac{a-b}{b-a} = -1$. Since the lines have slopes which are negative reciprocals of each other, they are perpendicular.

Exercise 15: The domain of f = range of f^{-1} = closed interval [-1,2] while range f = domain f^{-1} = closed interval [1/2, 4].

Exercise 18 : The domain of f = range of f^{-1} = closed interval [0,3] while range f = domain f^{-1} = closed interval [-1,1].



Exercise 15 Exercise 18

Graph of f is in blue and **graph of inverse is in red.**

II: Section 6.2: 2, 10, 15

Use $(\ln(g(x)))' = \frac{g'(x)}{g(x}$

Exercise 2: $f(x) = \ln(x^4 + 1)$ has $f'(x) = \frac{1}{x^4+1} \times (x^4 + 1)' = \frac{4x^3}{x^4+1}$

Exercise 10: $g(x) = \sqrt[3]{6x + 7} = (6x + 7)^{1/3}$ has $g'(x) = \frac{1}{3}(6x + 7)^{-2/3}(6) = \frac{2}{(6x+7)^{2/3}}$
Hence $f(x) = \ln \sqrt[3]{6x + 7}$ has $f'(x) = \frac{1}{(6x+7)^{1/3}} \times \frac{2}{(6x+7)^{2/3}} = \frac{2}{(6x+7)}$ Alternatively, use properties of logs: $\ln a^b = b \ln a$: $f(x) = \ln 6x + 7^{1/3} = \frac{1}{3} \ln 6x + 7$ so $f'(x) = \frac{1}{3} \, \frac{6}{6x+7}$

Exercise 15: $f(x) = \frac{1}{\ln x} + \ln \frac{1}{x} = (\ln x)^{-1} + \ln 1 - \ln x = (\ln x)^{-1} - \ln x$ which gives us

$$f'(x) = -1(\ln x)^{-2}(\ln x)' - \frac{1}{x} = \frac{-1}{x(\ln x)^2} - \frac{1}{x}.$$