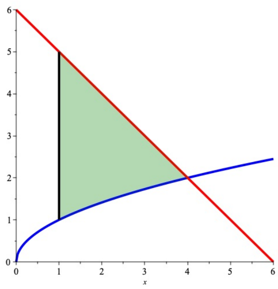
MATH 122: Calculus II

Some Notes on Assignment 5

I: Section 5.1: 2, 13, 23, 27

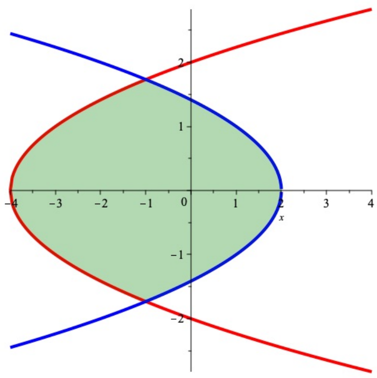
Exercise 2: The curves $x = 1$ and $y = \sqrt{x}$ intersect at (1,1), $x = 1$ and $x + y = 6$ intersect at (1,5), and $y = \sqrt{x}$ and $x + y = 6$ intersect at (2,4).
Slice the region into vertical slices so each slice goes from $y = \sqrt{x}$ to $y = 6 - x$. Area of region = $\int_1^4 (6 - x) - \sqrt{x}\, dx$.



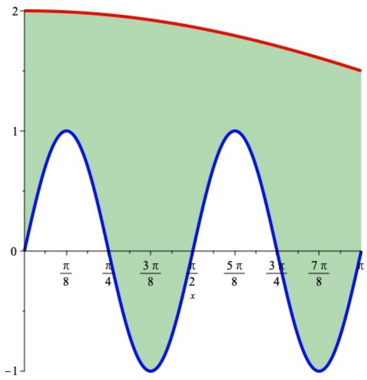
Exercise 2: $\boldsymbol{x = 1}$, $y = \sqrt{x}$, $x + y = 6$

Exercise 13: **The graph of $x = y^2 - 4$ is a parabola opening to the right** while **the graph of $x = 2 - y^2$ is a parabola opening to the left**. The curves intersect when $y^2 - 4 = 2 - y^2$; that is $2y^2 = 4 + 2 = 6$ so $y = \pm\sqrt{3}$ where $x = -1$. Imagine the region curved up into horizontal strips; Each strip between $y = -\sqrt{3}$ and $y = \sqrt{3}$ runs from the red curve to the blue curve. Hence the area is given by

$$\int_{-\sqrt{3}}^{\sqrt{3}} (2 - y^2) - (y^2 - 4)\, dy = \int_{-\sqrt{3}}^{\sqrt{3}} 6 - 2y^2\, dy = \left[6y - \frac{2}{3}y^3\right]_{-\sqrt{3}}^{\sqrt{3}} = 8\sqrt{3}$$



Exercise 13
$x = y^2 - 4$, $x = 2 - y^2$



Exercise 23
$y = \sin 4x$, $y = 1 + \cos \frac{x}{3}$

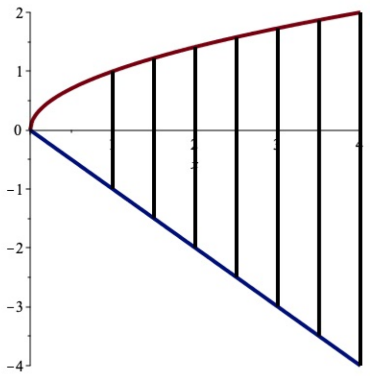
Exercise 23: Carve the region into vertical strips, reach running from $y = \sin 4x$ to $y = 1 + \cos \frac{x}{3}$ The area is

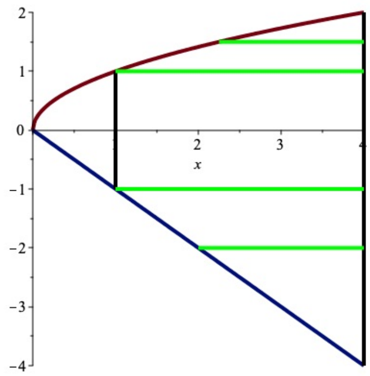
$$\int_0^{\pi} 1 + \cos(\frac{x}{3}) - \sin 4x\, dx = \left[x + 3\sin\frac{x}{3} + \frac{1}{4}\cos 4x\right]_0^{\pi} = (\pi + 3\sin\frac{\pi}{3} + \frac{1}{4}\cos 4\pi) - (0 + 3\sin\frac{0}{3} + \frac{1}{4}\cos 0)$$

which equals $(\pi + \frac{3}{2}\sqrt{3} + \frac{1}{4}) - (0 + 0 + \frac{1}{4}) = \pi + \frac{3}{2}\sqrt{3}$

Exercise 27 : We depict the region below. If we use vertical strips, each strip runs from the line $y = -x$ to the curve $y = \sqrt{x}$ so the area is $\int_{x=1}^{x=4} \sqrt{x} - (-x)\, dx = \int_{x=1}^{x=4} \sqrt{x} + x\, dx$ (See Figure 27a).
Slicing with horizontal strips shows (see Figure 27b) there are 3 kinds of strips. All end at $x = 4$ but start long different curves. For $y < -1$, the left end is on the line $x = -y$, for $-1 \le y \le 1$, the left end is on the vertical line $x = 1$, and for $y > 1$, the left end is on the parabola $x = y^2$. We can express the area of the entire region as the sum of 3 integrals:

$$\int_{y=-4}^{y=1} 4 - (-y)\, dy + \int_{y=-1}^{y=1} 4 - 1\, dy + \int_{y=1}^{y=2} 4 - y^2\, dy$$





II: Section 6.1: 1, 6, 11, 13

Exercise 1: For $f(x) = y = 3x+5$, solve for x in terms of y : $3x = y-5$ so $x = \frac{y-5}{3}$; hence $f^{-1}(x) = \frac{x-5}{3}$.

Exercise 6:Here $f(x) = y = \frac{4x}{x-2}$ so $(x-2)y = 4x$ or $xy - 2y = 4x$ which yields $xy - 4x = 2y$ or $x(y-4) = 2y$ so $x = \frac{2y}{y-4}$ and hence $f^{-1}(x) = \frac{2x}{x-4}$.

Exercise 11: Starting $f(x) = y = \sqrt[3]{x} + 1$, we have $y - 1 = \sqrt[3]{x}$ or $(y-1)^3 = x$ and $f^{-1}(x) = (x-1)^3$

Exercise 13: If $f(x) = y = ax + b$, then $y - b = ax$ and, since $a \neq 0$, $x = \frac{y-b}{a}$ so $f^{-1}(x) = \frac{x-b}{a}$.