MATH 122: Calculus II

Some Notes on Assignment 4

Page 407: 1, 13, 37, 45, 47, 50, 55, 57

Exercise 1: $\int_1^4 (x^2 - 4x - 3)\ dx = \left[\frac{x^3}{3} - 2x^2 - 3x\right]_1^4 = \left[\frac{64}{3} - (2 \times 16) - (3 \times 4)\right] - \left[\frac{1}{3} - 2 - 3\right] = -18$

Exercises 13:
Method I: $\int_{-1}^0 (2x + 3)^2\ dx = \int_{-1}^0 (4x^2 + 12x + 9)\ dx = \left[\frac{4}{3}x^3 + 6x^2 + 9x\right]_{-1}^0 = 0 - \left(-\frac{4}{3} + 6 - 9\right) = \frac{13}{3}$
Method II: Let $u = 2x+3$ so $du = 2dx$, $dx = \frac{1}{2}du$. Then $\int_{x=-1}^{x=0} (2x+3)^2 dx = \int_{u=1}^{u=3} \frac{1}{2}u^2 du = \left[\frac{u^3}{6}\right]_1^3 = \frac{13}{3}$

Exercise 37: $\int_0^\pi \sec^2 x\ dx = [\tan x]_0^\pi = \tan\pi - \tan 0 = 0 = 0$ is **not** a valid argument. We can't apply the Fundamental Theorem of Calculus here because the function $f(x) = \sec^2 x$ is not continuous on the interval $[0, \pi]$; it is not even defined at $\pi/2$. Observe also that $\sec^2 x$ is positive at most values of x in the interval and is never negative so its integral can not be 0.

Exercise 45: Since $\int_0^3 \sqrt{x^2 + 16}\, dx$ is a constant, its derivative with respect to x to 0.

Exercise 47: $\frac{d}{dx} \int_0^x \frac{1}{t+1} dt = \frac{1}{x+1}$ (Theorem 4.35)

Exercise 50: Average rate of change of f on $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$ while the average value of f' is $\frac{1}{b-a} \int_a^b f'(x)dx = \frac{1}{b-a}(f(b) - f(a)) = \frac{f(b)-f(a)}{b-a}$.

Exercise 55: Let F be any antiderivative of f. By the Fundamental Theorem of Calculus, $\int_a^{g(x)} f(t)\ dt = F(g(x)) - F(a)$. Note that $F(a)$ is just some constant. Then

$$\frac{d}{dx} \int_a^{g(x)} f(t)\ dt = \frac{d}{dx} \left(F(g(x)) - F(a)\right) = \text{ (Chain Rule) } F'(g(x)g'(x) - 0 = f(g(x)g'(x)$$

Exercise 57: We want $\frac{d}{dx} \int_2^{x^4} \frac{t}{\sqrt{t^3+2}}\, dt$. Use the result of Exercise 55 with $g(x) = x^4$ and $f(t) = \frac{t}{\sqrt{t^3+2}}$ to obtain the derivative as $f(x^4)g'(x) = \frac{x^4}{\sqrt{(x^4)^3+2}} 4x^3 = \frac{4x^7}{\sqrt{x^{12}+2}}$