MATH 122: Calculus II

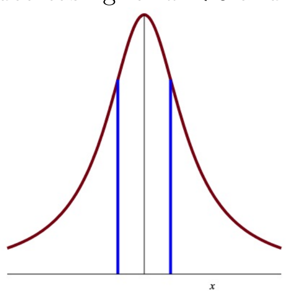
Some Notes on Assignment 3

Page 337: 1, 6,8, 11, 23, 24, 37, 40

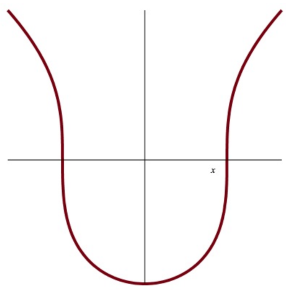
Exercise 1: $f(x) = -x^2 + 6x - 8$ on [1,6]
Here $f'(x) = -2x + 6$ and $f''(x) = -2$. Since the second derivative is always negative, the graph of f is concave down. There will be a local maximum where the first derivative is zero; that happens at $x = 3$. Other possible locations for extrema are the endpoints, 1 and 6, of the interval. Since $f(6) = -8 < f(1) = -3 < f(3) = 1$, there is a maximum at $x = 3$ and a minimum at $x = 6$.

Exercises 6 and 11: $f(x) = \frac{1}{x^2+1}$ has $f'(x) = \frac{-2x}{(x^2+1)^2}$ which is positive for $x < 0$ and negative for $x > 0$. Hence f is increasing on the negative numbers and decreasing on the positives. There is a maximum value of 1 at $x = 0$.
The second derivative if $f''(x) = \frac{6x^2-2}{(x^2+1)^3}$; Since the denominator is positive, the sign of f'' depends on the sign of the numerator. Now $6x^2 - 2$ is negative for $-\sqrt{3}/3 < x < \sqrt{3}/3$ and otherwise positive. The graph of f is concave up for $x < -\sqrt{3}/3$ and $x > -\sqrt{3}/3$ and concave down between these 2 values at which there are points of inflection.

8: $f(x) = \sqrt[3]{x^2-9} = (x^2-9)^{1/3}$ has $f'(x) = \frac{2x}{3\sqrt[3]{(x^2-9)^2}}$ which changes sign from negative to positive at $x = 0$ so f is decreasing for $x < 0$ and increasing for $x > 0$



Graph of $f(x) = \frac{1}{x^2+1}$
Vertical lines indicate location of inflection points



Graph of $f(x) = \sqrt[3]{x^2-9}$

Exercise 23: $f(x) = x^3 + x^2 + x + 1$ on [0,4]. We have $f(0) = 1, f(4) = 85$ and $\frac{f(4)-f(0)}{4-0} = 21$, the slope of the line between the endpoints on the graph. The derivative $f'(x) = 3x^2 + 2x + 1$ has value 21 when $x = \frac{-1\pm\sqrt{61}}{3}$ but only $x = \frac{-1+\sqrt{61}}{3}$ is in the interval [0,4].

24: 1 hour, 40 minutes = (1 + 2/3) = 5/3 hours so the average velocity of the trip was $\frac{125}{5/3} = 75$ mi/hr. By the Mean Value Theorem, there must be an instant when the car had instantaneous velocity of 75 mi/hr which is above the speed limit of 65 mi/hr. See the discussion of Example 4 in Section 3.2.

37: Cost of producing x calculators is $C(x) = 500 + 6x + 0.02x^2$. If x calculators are sold at \$18 each, then
(a) Revenue = $R(x) = 18x$ and (b) $P(x)$ = profit = revenue - cost =$18x - (500 + 6x + 0.02x^2)$
(c) $P'(x) = 18 - 6 - .04x$ and $P''(x) = -.04$ so profit is maximized when $P'(x) = 0$; that is $12 = .04x$ which occurs when $x = 300$.
(d) Maximum Profit = $P(300)$ = \$1300.

40: Let $p(t)$ be distance moved t seconds after engaging the brake and $v(t)$ the velocity at time t. Let S be the number of seconds it takes to slow down from 44 ft/sec to 32 ft.sec. Then $p(0) = 0, p(S) = 114, v(0) = 44, v(S) = 32$ and since deceleration is constant, we have $v(t) = 44 - \frac{12}{S}t$ because velocity drops 12 ft/sec in S seconds. From the velocity function, we derive $p(t) = 44t - \frac{6}{S}t^2$ since $p(0) = 0$. We are given $114 = p(S) = 44S - \frac{6}{S}S^2 = 44S - 6S = 38S$ so $S = 114/38 = 3$.

(a) It takes 3 seconds to reduce the speed to 32 ft/sec.
(b) Deceleration = $\frac{44-32}{3} = 4ft/sec^2$.
(c) Sled comes to a stop when $v = 0$: We now know $v(t) = 44 - 4t$ so it takes 11 seconds to reach a full stop at which the sled has traveled $p(11) = 44(11) - 2(11^2) = 242$ feet.