MATH 122: Calculus II
Some Notes on Assignment 15

I: Section 7.1: 37, 48, 53

Exercise 37: Use integration by parts on $\int \arccos x\, dx$ with $u = \arccos x$, $dv = dx$. The $du = \frac{-1}{\sqrt{1-x^2}}\, dx$ and $v = x$. Then $\int \arccos x\, dx = uv - \int v\, du = x \arccos x - \int \frac{-x}{\sqrt{1-x^2}} = x \arccos x - \sqrt{1-x^2} + C$

Exercise 48: Work $= \int_0^1 x^5\sqrt{x^3+1}\, dx$. Write integrand as $(x^3)(x^2(x^3+1)^{1/2})$ and use integration by parts with $u = x^3, du = 3x^2\, dx,\ dv = x^2(x^3+1)^{1/2}\, dx,\ v = \frac{2}{9}(x^3+1)^{3/2}$
Then $F(x) = \int x^5\sqrt{x^3+1}dx = (x^3)\frac{2}{9}(x^3+1)^{3/2} - \int \frac{2}{9}(x^3+1)^{3/2}(3x^2)dx = \frac{2}{9}x^3(x^3+1)^{3/2} - \frac{4}{45}(x^3+1)^{5/2}$
so Work $= F(1) - F(0) = [\frac{2}{9}(2^{3/2}) - \frac{4}{45}2^{5/2}] - [0 - \frac{4}{45}] = \frac{4}{45}\left(\sqrt{2}+1\right)$

Exercise 53: The argument is correct up to the statement $\int \frac{1}{x}\, dx = 1 + \int \frac{1}{x}\, dx$, but then it treats $\int \frac{1}{x}\, dx$ as if it were a **number** when it fact it is a set of functions each with the same derivative. A correct next line would be $\ln x + C_1 = \ln x + C_2$ where C_1 and C_2 are any constants which need not be equal.

II: Section 7.2: 13, 17, 21

Exercise 13: $\int \tan^6 x\, dx$
. We'll use the identity $\tan^2 x = \sec^2 x - 1$ and $(\tan x)' = \sec^2 x$:
$\tan^6 x = (\tan^4 x)(\tan^2 x) = (\tan^4 x)(\sec^2 x - 1) = (\tan^4 x)(\sec^2 x) - \tan^4 x = (\tan^4 x)(\sec^2 x) - (\tan^2 x)(\tan^2 x) = (\tan^4 x)(\sec^2 x) - (\tan^2 x)(\sec^2 x - 1) = (\tan^4 x)(\sec^2 x) - (\tan^2 x)(\sec^2 x) + \tan^2 x = (\tan^4 x)(\sec^2 x) - (\tan^2 x)(\sec^2 x) + \sec^2 x - 1 = (\tan^4 x)(\tan x)' - (\tan^2 x)(\tan x)' + (\tan x)' - 1$. Thus $\int \tan^6 x\, dx = \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x - x + C$.

Exercise 17: $\int (\tan x + \cot x)^2\, dx = \int \tan^2 x + 2(\tan x)(\cot x) + \cot^2 x\, dx = \int \tan^2 x + 2 + \cot^2 x\, dx$
$= \int (\tan^2 x + 1) + (\cot^2 x + 1)\, dx = \int \sec^2 x + \csc^2 x\, dx = \tan x - \cot x + C$

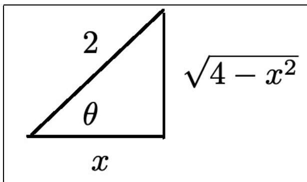
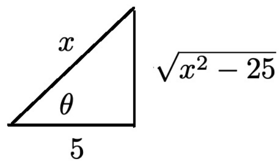
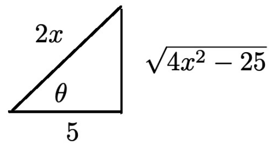
Exercise 21: Observe that $\sin^3 x = (\sin^2 x)(\sin x) = (1 - \cos^2 x)(\cos x)'$ which leads to the substitution $u = \cos x, -du = \sin x\ dx$. When $x = 0, u = 1$ and when $x = \pi/4, u = \sqrt{2}/2$. Thus $\int_0^1 \sin^3 x\, dx = \int_1^{\sqrt{2}/2}(1-u^2)(-1)\, du = \left[\frac{u^3}{3} - u\right]_1^{\sqrt{2}/2} = [\frac{1}{3}\frac{2\sqrt{2}}{8} - \frac{\sqrt{2}}{2}] - [\frac{1}{3} - 1] = -\frac{5\sqrt{2}}{2} + \frac{2}{3}$

III: Section 7.3: 1, 6, 10

Exercise 1: $\int \frac{1}{x\sqrt{4-x^2}}\,dx$. See triangle below where 2 is the hypotenuse and x the adjacent side, making $\sqrt{4-xx^2}$
the opposite side. The simplest ration involving x is $x/2$ which is $\cos\theta$. [If you make x the opposite side, then $x/2 = \sin\theta$; that substitution will lead to a slightly different looking, but equivalent answer]. With $x/2 = \cos\theta$, we have $x = 2\cos\theta$, $dx = -2\sin\theta\,d\theta$, $\sqrt{4-x^2} = 2\sin\theta$. With these substitutions, we have

$$\int \frac{1}{x\sqrt{4-x^2}}\,dx = \int \frac{1}{(2\cos\theta 2)(\sin\theta)}(-2\sin\theta)\,d\theta = -\frac{1}{2}\int \sec\theta\,d\theta = -\frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

$$= -\frac{1}{2}\ln\left|\frac{2}{x} + \frac{\sqrt{4-x^2}}{x}\right| + C = -\frac{1}{2}\ln\left|\frac{2+\sqrt{4-x^2}}{x}\right| + C$$

		
Exercise 1: $\int \frac{1}{x\sqrt{4-x^2}}\,dx$	Exercise 6: $\int \frac{1}{x^3\sqrt{x^2-25}}\,dx$	Exercise 10 : $\int \frac{1}{\sqrt{4x^2-25}}\,dx$

Exercise 6: $\int \frac{1}{x^3\sqrt{x^2-25}}\,dx$
See triangle above with x as hypotenuse, 5 as adjacent side, and $\sqrt{x^2-25}$ as opposite side. The substitutions are $\sec\theta = \frac{x}{5}, x = 5\sec\theta, dx = 5(\sec\theta)(\tan\theta)\,d\theta, \sqrt{x^2-25} = 5\tan\theta$. These yield

$$\int \frac{1}{x^3\sqrt{x^2-25}}\,dx = \int \frac{5(\sec\theta)(\tan\theta)}{5^3(\sec^3\theta)(5\tan\theta)}\,d\theta = \frac{1}{5^3}\int \frac{1}{\sec^2\theta}\,d\theta = \frac{1}{5^3}\int \cos^2\theta\,d\theta = \frac{1}{5^3}\int \frac{1+\cos 2\theta}{2}\,d\theta$$

$$= \frac{1}{250}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C = \frac{1}{250}(\theta + \sin\theta\,\cos\theta) + C = \frac{1}{250}\left(\operatorname{arcsec}\left(\frac{x}{5}\right) + (\frac{\sqrt{x^2-25}}{x})(\frac{5}{x})\right) + C$$

Exercise 10: $\int \frac{1}{\sqrt{4x^2-25}}\,dx$. See triangle above with $2x$ as hypotenuse, 5 as adjacent side, and $\sqrt{4x^2-25}$ as opposite side. The substitutions are $\sec\theta = \frac{2x}{5}, x = \frac{5}{2}\sec\theta, dx = \frac{5}{2}(\sec\theta)(\tan\theta)\,d\theta, \sqrt{4x^2-25} = 5\tan\theta$. These yield

$$\int \frac{1}{x^3\sqrt{x^2-25}}dx = \int \frac{\frac{5}{2}(\sec\theta)(\tan\theta)}{5(\tan\theta)}d\theta = \frac{1}{2}\int \sec\theta d\theta = \frac{1}{2}\ln|\sec\theta + \tan\theta| + C = \frac{1}{2}\ln\left|\frac{2x+\sqrt{4x^2-5}}{5}\right| + C$$