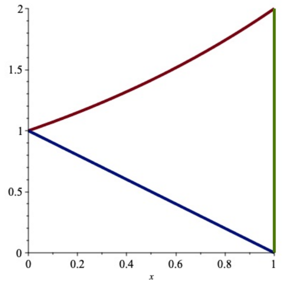
MATH 122: Calculus II

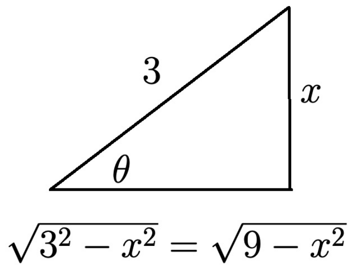
Some Notes on Assignment 11

I: Section 6.5: 45, 48, 53

Exercise 45 : The region bounded by $y = 2^x, y = 1 - x, x = 1$ is shown below. Each vertical slice runs from $x + y = 1$ to $y = 2^x$. The area is $\int_0^1 2^x - (1 - x)\, dx$



Exercise 45: Curves intersect at (0,1), (1,0) and (1,1)



Section 6.7: Exercise 18

Exercise 48: The amount accumulated by time t is $A(T) = \int_0^T 5(.95^t)dt = \left[\frac{5}{\ln .95}.95^t\right]_0^T = \frac{5}{\ln .95}\left[.95^T - 1\right]$. We want to choose T so that this amount is 50: $50 = \frac{5}{\ln .95}\left[.95^T - 1\right]$. We have $10 \ln .95 = .95^T - 1$ so $.95^T = 1 + 10 \ln .95$ which makes $T = \frac{\ln(1+10\ln .95)}{\ln .95} \approx 14.02$ minutes.

Exercise 53: (a) $R(x) = a \log\left(\frac{x}{x_0}\right)$ so $R(x_0) = a \log\left(\frac{x}{x_0}\right) = a \log 1 = a(0) = 0$

(b) We can also write $R(x)$ as $R = a \log\left(\frac{x}{x_0}\right) = a \log x - a \log x_o$ where second term is constant. Then $S(x) = dR/dx = \frac{a}{\ln 10}\frac{1}{x} = \frac{k}{x}$ where $k = \frac{a}{\ln 10}$ is constant; Thus S is inversely proportional to x.
Finally, note $S(2x) = \frac{k}{2x} = \frac{1}{2}\frac{k}{x} = \frac{1}{2}S(x)$ so $S(x) = 2S(2x)$.

II: Section 6.6: 10, 15, 19

Exercise 10: Let $N(t)$ be the number of ticks per minute t days after it was 2000. . Then $N(t) = 2000e^{rt}$. With $N(10) = 1500$, we have $2000e^{10r} = 1500$ or $e^{10r} = \frac{3}{4}$ so $10r = \ln(3/4)$ and $r = \frac{\ln(3/4)}{10}$. Thus $N(t)2000e^{(t/10)\ln(3/4)} = 2000e^{\ln(3/4)^{t/10}} = 2000\left(\frac{3}{4}\right)^{t/10}$.
Half life is at time T where $N(T) = 1000$ so $2000\left(\frac{3}{4}\right)^{T/10} = 1000$ which gives $\left(\frac{3}{4}\right)^{T/10} = \frac{1}{2}$. Solve T to get $T/10 \ln(3/4) = \ln(1/2)$ so $T = \frac{10\ln(1/2)}{\ln(3/4)} \approx 24.09$ days

Exercise 15: Let $U(t)$ be number of units of the drug t hours after the operation. Then $U(t) = U_0e^{rt}$. We want $U(3/4) = (20)(30) = 600$. Thus $U_0 = 600e^{-3r/4}$ but since half-life is 4 hours, $1/2 = e^{4r}$ so $r = \frac{\ln(1/2)}{4} = \frac{-\ln 2}{4}$. Thus

$$U_0 = 600e^{-(\frac{3}{4})(-\frac{\ln 2}{4})} = 600e^{\frac{3}{16}\ln 2} = 600e^{\ln(2^{3/16}} = 600\left(2^{\frac{3}{16}}\right) \approx 683.27$$

Exercise 19: Let $q(t)$ be amount of ^{14}C present at t years. Then $q(t) = q_0e^{at}$. Since half-life is 5700 years, we have $\frac{1}{2}q_0 = q_o e^{5700a}$. Thus $5700a = \ln 1/2 = -ln2$ so $a = \frac{-\ln 2}{5700}$. We are given that $(1/5)q_0 = q_0e^{aT}$ for some T. Hence $aT = \ln 1/5 = -ln5$ so $T = \frac{-(\ln 5)5700}{-\ln 2} \approx 13{,}235$ years.

III: Section 6.7: 1, 9, 18

Exercise 1: (a) $\arcsin(-\sqrt{2}/2) = -\pi/4$; (b) $\arccos(-1/2) = 2\pi/3$; (c) $\arctan(-\sqrt{3}) = -\pi/3$

Exercise 9: (a) $\sin(\arctan\sqrt{3}) = \sin(\pi/3) = \sqrt{3}/2$ (b) $\cos(\arcsin 1) = \cos(\pi/2) = 0$;
(c) $\tan(\arccos 0) = \tan(\pi/2)$ which is undefined..

Exercise 18: To find $\sec(\arcsin x/3)$, draw a right triangle with angle θ that has $\sin\theta = x/3$. Let opposite side by x and hypotenuse 3. Find adjacent third side by Pythagorean Theorem. Then $\sec\theta = \frac{opposite}{adjacent} = \frac{3}{\sqrt{9-x^2}}$. See figure above.