MATH 122: Calculus II

Some Notes on Assignment 1

Measuring Inequality: 2, 3, 4.

2: Find the Gini index corresponding to the Lorenz curve $f(x) = x^3$.

Solution: Gini Index $= 2\int_0^1 x - x^3 dx = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 2\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{2} = .5$

3: Find the Gini index corresponding to the Lorenz curve $f(x) = \frac{x}{4} + \frac{3}{4}x^3$.

Solution: Gini Index $= 2\int_0^1 x - \left(\frac{x}{4} + \frac{3}{4}x^3\right) dx = 2\int_0^1 \frac{3}{4}x - \frac{3}{4}x^3 dx = \frac{3}{2}\int_0^1 x - x^3 dx = \left(\frac{3}{2}\right)\left(\frac{1}{4}\right) = \frac{3}{8}$, using our answer from Exercise 2.

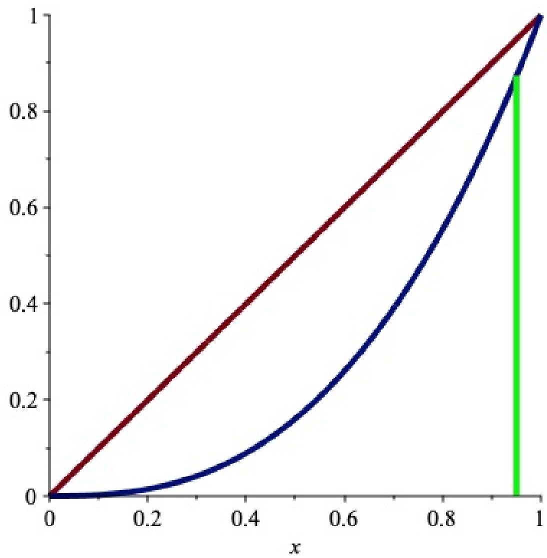
4: The CIA website reports the Gini index for the distribution of family income in the United States to be .45.

(a) Determine the number p so that the Gini index is .45 if the Lorenz curve has the form of a power function $f(x) = x^p$.

(b) According to this model, how much of the family income is earned by the top 5% of families?

Solution: (a): $.45 = \frac{9}{20} = 2\int_0^1 x - x^p dx = 2\left[\frac{x^2}{2} - \frac{x^{p+1}}{p+1}\right]_0^1 = 1 - \frac{2}{p+1}$ so $\frac{2}{p+1} = 1 - \frac{9}{20} = \frac{11}{20}$ and hence $\frac{p+1}{2} = \frac{20}{11}$ and $11p + 11 = 40$ or $p = \frac{29}{11}$.

(b): Bottom 95% earn $.95^{\frac{29}{11}} \approx 87.4\%$ so top 5% earn about $100 - 87.4 = 12.6$ percent.



The Lorenz curve $f(x) = x^{\frac{29}{11}}$, the perfect equality" line $y = x$ and the vertical line $x = .95$ shows that the bottom 95% earn a bit above 87%of the total income.