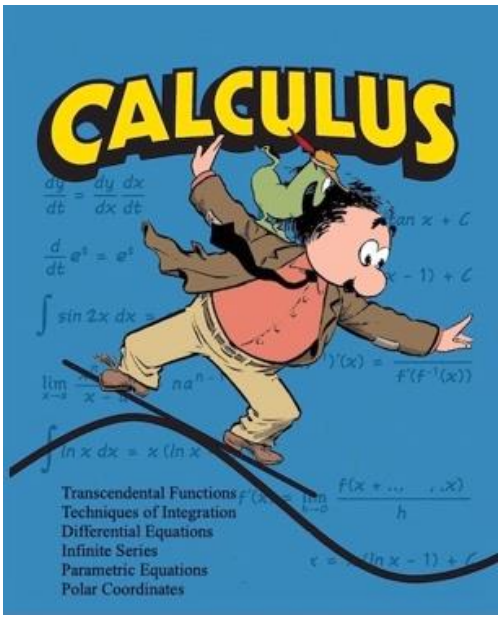
Calculus II

MATH 122

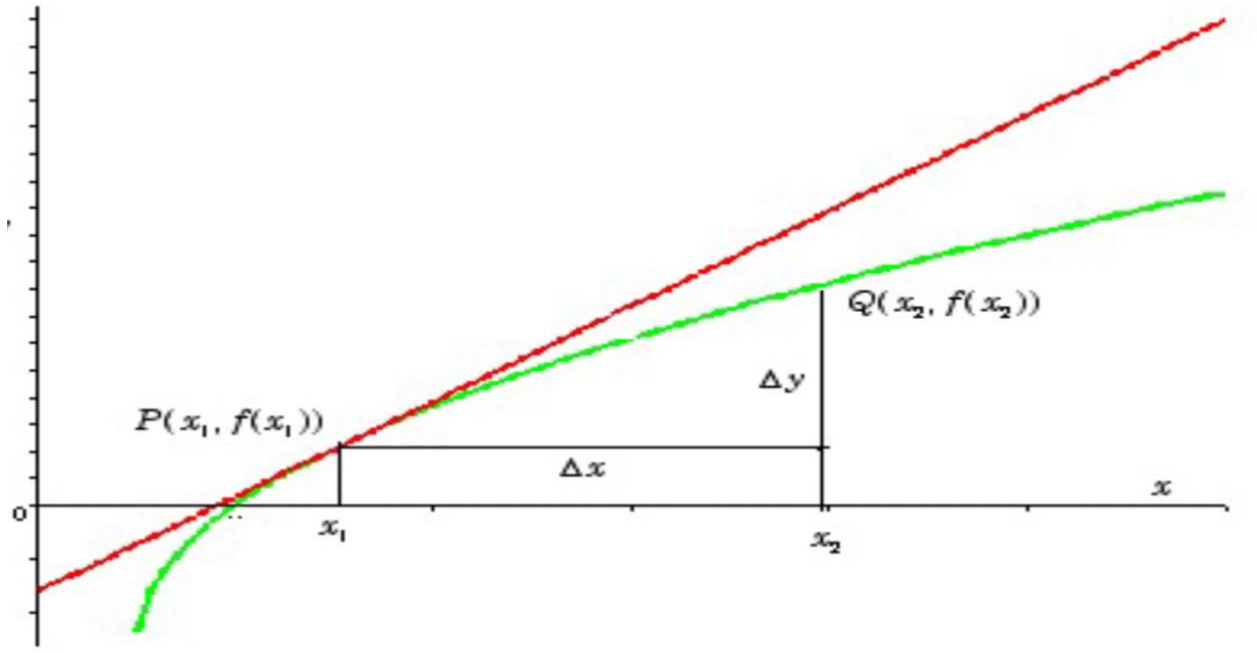
Michael Olinick

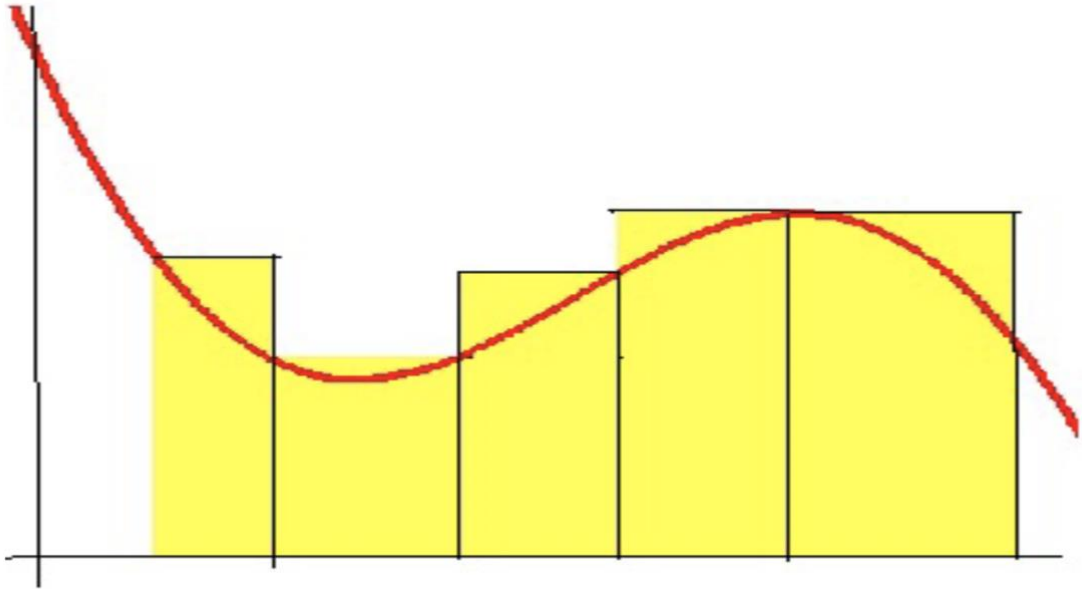


Fall 2025

Contents

The Two Most Important Diagrams in Calculus





MATH 122 C and D Calculus II

Course Description
Fall Term 2025

Course Title: **Calculus II**

Description: A continuation of MATH 121 – Calculus II, may be elected by students who have had an introduction to analytic geometry and calculus in secondary school.

Topics include: a brief review of limits and the Fundamental Theorem of Calculus, sequences and series, natural logarithm and exponential functions, elementary transcendental functions, techniques of integration, power series and Taylor's theorem, applications of integrals including economics and probability, polar coordinates, improper integrals, ordinary differential equations.

Successful completion of this course satisfies the "Deductive Reasoning and Analytical Processes" distribution requirement.

Learning Outcomes: In this course, you will

- solve many types of integrals, including those with trigonometric functions and infinite bounds and domains
- explore infinite sequences and series and their convergence and limits
- learn to sit with hard mathematics and approach tough problems one step at a time
- become familiar with interdisciplinary applications of mathematics
- learn to write clean and concise solutions to mathematical problems

Instructor: Michael Olinick (*molinick@middlebury.edu*), 202 Warner, 443-5559. Home: 388-4290. Office Hours: Monday, Wednesday and Friday : 9:35-11 AM; Monday and Wednesday:12:10 – 1 PM; Thursday: 10 - 11. **I am happy to make an appointment to see you at other mutually convenient times.**

Meeting Times: 122C:11:15 AM– 12:05 PM Monday, Wednesday and Friday in Warner 100; 11:15 –12:30 PM on Thursday in Warner 100.

122D: 1:10 PM– 2:00 PM Monday, Wednesday and Friday in Warner 100; 12:45 –2:00 PM on Thursday in Warner 100.

Course Websites:
http://f25.middlebury.edu/MATH0122C or :
http://f25.middlebury.edu/MATH0122D

Prerequisites: A good background is a year's study of calculus in secondary school or completion of MATH 121: Calculus I at Middlebury. If most of the material in Chapters 1 -

5 of our text looks familiar, then you should begin the term in MATH 122. We realize that you probably haven't done much calculus in the past four months. Your differentiating and integrating skills will be a bit rusty at first, but they should soon be back in smooth working order. You can always transfer to Calculus I after one or two weeks if this proves advisable. Please note that you can not enroll in this course if you have received a 4 or a 5 on the Advanced Placement BC Calculus Exam; students with this background should sign up for MATH 200: *Linear Algebra.*

Textbook: Earl Swokowski, Michael Olinick, and Dennis Pence, *Calculus of a Single Variable Calculus* (2nd Edition) or *Calculus* (6th Edition). Your daily assignments will include a few pages of reading in the text. Be certain to read the book carefully (with pencil and paper close by!) and to complete the relevant reading before coming to class and before embarking on the homework problems. The text contains answers to odd-numbered problems. You should be able to pick up a copy at a nominal price. There are copies on reserve at the Davis Family Library. You can find links to digital versions of the chapters on the course website.

Requirements: There will be three midterm examinations and a final examination in addition to required daily homework assignments and, perhaps, an occasional very short paper. The midterm examinations will be given in evening outside normal class meetings to eliminate time pressure. Tentative dates for these tests are:

Monday, October 6
Monday, November 3
Wednesday, December 3

The College's Scheduling Officer has tentatively set the times and dates of the Final Exam as

9 AM to Noon on Thursday, December 11 for MATH 122C
7 PM to 10 PM on Friday, December 12 for MATH 122D

Course Grades: Each of the three midterm exams will be worth approximately 20%, the final about 30%, team projects roughly 10%. I will make adjustments with later work counting more heavily if students show improvement over earlier results.

The mathematics department regards a C grade as an indication of satisfactory *understanding* of the course material, a B as good/very good understanding and an A as an excellent/superior grasp of the material. Typically, but not always, these tend to correlate with averages in the 70's, 80's and 90's, respectively. I do strive to issue course grades keeping in mind the hundreds of students I have had in calculus classes over the years.

Accommodations: Students who have *Letters of Accommodation* in this class are encouraged to contact me as early in the semester as possible to ensure that such accommodations are implemented in a timely fashion. For those without *Letters of Accommodation*, assistance is available to eligible students through the Disability Resource Center. Please contact ADA Coordinators Jodi Litchfield or Peter Ploegman for more information: Peter Ploegman can be reached at pploegman@middlebury.edu or 802-443-2382 and Jodi Litchfield can

be reached at litchfie@middlebury.edu or 802-443-5936. All discussions will remain confidential.

Homework: Mathematics is not a spectator sport! You must be a participant. The only effective way to learn mathematics is to do mathematics. In your case, this includes working out several hundred calculus problems.

There will be daily written homework assignments which you will be expected to complete and submit. They will be corrected and assigned a numerical score, but I view these assignments primarily as **learning** rather than testing experiences. I will occasionally assign some challenging problems which everyone may not be able to solve. You should, however, make an honest attempt at every problem.

Each homework assignment will probably take you between 2 and 3 hours to complete; this includes the reading and problem solving. If you keep pace with the course by spending an hour or so each day on it, then you will be quite successful. If you wait until the end of the week and then try to spend one 6 hour block of time on the material, then experience shows you face disaster!

One of the essential characteristics of college life that distinguishes it from secondary school is the increased responsibility placed on *you* for your own education. **Most of what you will learn will not be told to you by a teacher inside a classroom**. Even if our model of you were an empty vessel waiting passively to be filled with information and wisdom, there wouldn't be time enough in our daily meetings to present and explain it all. We see you, more appropriately, as an *active* learner ready to confront aggressively the often times subtle and difficult ideas our courses contain. You will need to listen and

Grades: Grades in the course will be based primarily on the examinations, team projects and class participation; effort and success on the homework will be considered in border line situations

Help: Please see me **immediately** if you have any difficulties with this course. There are ample resources on campus for assistance. One regularly scheduled event are the "Drop In" tutoring sessions on Sunday, Tuesday, Wednesday and Thursday evenings from 7 to 9 PM in the Quantitative Center at Bi-Hall. These sessions should begin by this Thursday.

Academic Integrity: As an academic community devoted to the life of the mind, Middlebury requires of every student complete intellectual honesty in the preparation and submission of all academic work. Details of our Academic Honesty, Honor Code, and Related Disciplinary Policies are available in Middlebury's *Handbook*.

Honor Code: The four examinations are to be taken without any information of any kind about the exam or its questions from any source but me. (I will give you a clear idea about what to expect on each exam.) Exams will be closed book, closed notes, and taken **without calculators**. Any departure from these policies would be a violation of the Honor Code and

thus would be subject to a judicial review.

Homework is a different story: Calculators are o.k. for checking work and for doing tedious calculations. You may make use of a free version of Desmos, for example, for graphing and visualizing functions and their derivatives/integrals. I encourage you to discuss problems with other students. But when you do that you must give credit for another's contributions by naming the person on your solution. **Writing up** each problem solution must be done by you only. Failure to follow these requirements is a violation of the Honor system. The principle here is simple: consultation and collaboration are welcome, but you must explicitly acknowledge **any and all** intellectual content that isn't solely yours. This rule should apply in **all** of your work at Middlebury College.

Generative AI in Our Class

Here is an image depicting Isaac Newton and Gottfried Wilhelm Leibniz in a heated feud over the creation of calculus. The scene captures the tension between the two historical figures, surrounded by symbols of their mathematical contributions**.** (Generated by ChatGPT4)

AI Use Policy for Our Calculus Course: This policy aims to foster a productive and ethical learning environment where students can use AI to support their education while maintaining the integrity of their work

1. *Purpose of AI Tools*: - AI tools can be valuable for enhancing learning and understanding complex calculus concepts. I encourage students to use AI for study aids, explanations, and practice problems to supplement their learning.
2.

2. *Permissible Uses*

• Study Assistance: Students may use AI tools such as calculators, solvers, or educational platforms to check your work, understand solutions, and explore additional problems.

• Concept Clarification: AI can be used to clarify concepts, get alternative explanations, or see step-by-step solutions to practice problems.

• Preparation for Exams: Students may use AI to generate practice problems, review materials, and study guides.

3. *Prohibited Uses*:

•Assignment Completion: Students are prohibited from using AI to complete assignments, quizzes, or exams. This includes using AI to solve problems directly, generate answers, or perform any task that contributes to a graded assessment.

• Plagiarism: Using AI to generate written content (*e.g.,* essays, explanations) that is submitted as original work without proper attribution is considered plagiarism and will be treated as an academic integrity violation. (Along those lines…I used ChatGPT 4 to write 90% of this AI policy.)

• Communication During Exams: AI tools must not be used during exams or quizzes unless explicitly allowed. This includes AI-powered calculators, chatbots, or any online resources.

4. *Transparency and Attribution*:
• When AI tools are used to aid in understanding or problem-solving, students should acknowledge the use of these tools. This includes citing the tool in written work if it contributes significantly to a solution or explanation.

5. *Consequences of Misuse*:
• Misuse of AI tools, particularly in ways that violate academic integrity, will result in disciplinary action according to Middlebury's policies. Consequences may include receiving a zero on the assignment, failing the course, or further disciplinary measures.

6. *Encouragement of Ethical Use*:
•I encourage students to use AI ethically to enhance your earning experience. The goal is to use these tools to develop a deeper understanding of calculus, not to bypass the learning process.

7. *Instructor's Discretion*:
• I reserve the right to modify this policy as needed and will communicate any changes to the class promptly. Any use of AI tools not explicitly covered by this policy should be discussed with me.

Citation for my use of AI to generate this section
"Write a policy on the use of AI for my calculus course" prompt. *ChatGPT* 4.0, OpenAI, August 27, 2024, chat.openai.com/chat.

Review Calculus I (1 week)

Limits, Continuity, Derivatives
Riemann Sums and Fundamental Theorem of Calculus
Integrals and *u*-Substitution

Transcendental Functions (3 weeks)

Derivative of Inverse Function
Logarithmic and Exponential Functions
Separable Differential Equations/ Laws of Growth and Decay
Inverse Trigonometric Functions
Indeterminate Forms and l'Hôpital's Rules

Techniques of Integration (2 weeks)

Integration By Parts
Trigonometric Integrals
Trigonometric Substitution
Partial Fraction Decomposition
Improper Integrals and Their Applications

Infinite Sequences and Series (3 weeks)

Sequences
Series
Positive-Term Series
Ratio and Root Tests
Alternating Series/Absolute Convergence
Tests for convergence
Power series
Maclaurin and Taylor series

Parametric Equations (1 week)

Parametric Equations
Arc Length
Polar Coordinates

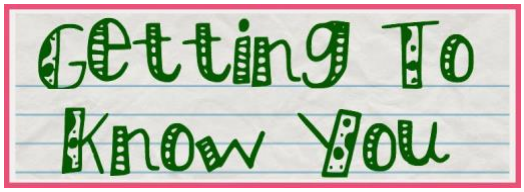
MATH 122 C & D: CALCULUS II SCHEDULE FALL 2025

Week Of:	Monday	Tuesday	Wednesday	Thursday	Friday
September 8	*FIRST DAY OF CLASS*		Assignment 0	No Class	No Class
September 15	Assignment 1		Assignment 2	Assignment 3	Assignment 4
September 22	Assignment 5		Assignment 6	Assignment 7	Assignment 8
September 29	Assignment 9		Assignment 10	No Class	Assignment 11
October 6	**EXAM 1**		Assignment 12	Assignment 13	*Midterm recess*
October 13	Assignment 14		Assignment 15		Assignment 16
October 20	Assignment 17		Assignment 18		Assignment 19
October 27	Assignment 20		Assignment 21		Assignment 22
November 3	**EXAM 2**		Assignment 23		Assignment 24
November 10	Assignment 25		Assignment 26		Assignment 27
November 17	Assignment 28		Assignment 29		Assignment 30
November 24	*Thanksgiving Recess*		*Thanksgiving Recess*	*Turkey Day*	*Thanksgiving Recess*
December 1	Assignment 31		**EXAM 3**		Assignment 32
December 8	Assignment 33			**122C Final 9 -Noon**	**122D Final 7 – 10 PM**

Final Examination: The College's Scheduling Officer has set the times and dates as

9 AM to Noon on Thursday, December 11 for MATH 122C
7 PM to 10 PM on Friday, December 12 for MATH 122D

Assignment 0
(Adapted from *Homework 0* by Alex Lyford)
Due: Wednesday, September 10



Reading

Read carefully through the materials in the First Day Packet.

Writing

You may submit an electronic copy of this assignment to me (molinick@middlebury.edu) with the subject line: **MATH 122 Assignment 0** or print it out and bring it to Wednesday's class. **Make sure you include your name at the top of the document.**

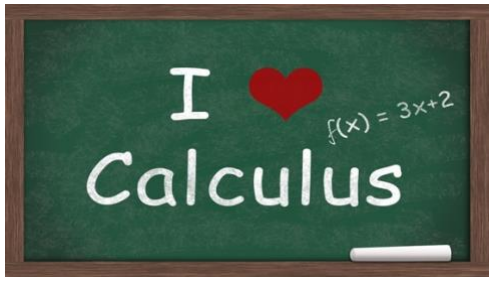
Your task is to create a document describing yourself, your goals, and what you hope to get out of our Differential Equations class. Please provide your name at the top of the first page along with your major or likely major and your anticipated graduate date.

Start with an autobiographical statement about yourself that will help me to get to know a little about you. Where did you grow up? Why did you come to Middlebury? What are your likes and dislikes? Do you have any hobbies that you do regularly? Do you have a major extracurricular activity such as athletics, theatre or *The Campus?*

After the biographical statement tell me about your mathematical, statistical, and computer programming background. What did you like about previous mathematics, statistics, and/or programming classes? What did you dislike? What aspects did you find easy? What aspects did you find challenging?

The next part should discuss your plans for the remainder of your time in college, and what you hope to do after you graduate. Is more schooling the next step, or do you plan to get a job? It's okay to not have any idea what you want to do after graduation, but list some possibilities so that I can better tailor the materials in class to your potential career opportunities.

Finally tell me about your thoughts and expectations for this class. What are you hoping and/or expecting to learn? What do you think the challenges of this course might be? What, if anything, have you heard about this course from your peers? What expectations do you have of me? Feel free to also discuss anything I've failed to ask here!



Assignments

Assignment	*Reading*	*Exercises*
1	Measuring Inequality	Measuring Inequality: 2, 3, 4
2	Review Chapter 1, 2	Measuring Inequality: 5, 6 Page 242: 2, 9,21, 33, 46, 52, 64, 80
3	Review Chapter 3	Page 337: 1, 6,8, 11, 23, 24, 37, 40
4	Review Chapter 4; Focus on Fundamental Theorem of Calculus	Page 407: 1, 13, 37, 45, 47, 50, 55, 57
5	Review Sections 5.1 and 6.1 Read Section 6.2	Section 5.1: 2, 13, 23, 27 Section 6.1: 1, 6, 11, 13
6	Review Section 6.2 Read Section 6.3	Section 6.1: 14, 15, 18 Section 6.2: 2, 10, 15
7	Review Section 6.3 Read Section 6.4	Section 6.1: 27, 35 Section 6.2: 26, 33, 37 Section 6.3: 1, 7, 14
8	Review Section 6.4 Read Section 6.5	Section 6.2: 44, 51 Section 6.3: 20, 25, 32 Section 6.4: 1, 7, 14
9	Review Section 6.5 Read Section 6.6	Section 6.3: 37, 47, 54 Section 6.4: 19, 25, 31 Section 6.5: 1, 8, 17
10	Review Section 6.6 Read Section 6.7	Section 6.4: 45, 48, 52 Section 6.5: 24, 31, 38 Section 6.6: 1, 5, 7
11	Review Section 6.7 Read Section 6.9	Section 6.5: 45, 48, 53Z Section 6.6: 10, 15, 19 Section 6.7: 1, 9,19
Assignment	*Reading*	*Exercises*
12	Review Section 6.9 Read Section 7.1	Section 6.6: 21, 22 Section 6.7: 27, 30, 41 Section 6.9: 1, 10, 19
13	Review Section 7.1 Read Section 7.2	Section 6.7: 51, 60, 69 Section 6.9: 28, 36, 42

		Section 7.1: 1, 7, 13
14	Review Section 7.2 Read Section 7.3	Section 6.9: 49, 51, 60, 82 Section 7.1: 19, 24, 31 Section 7.2: 1, 5, 9
15	Review Section 7.3 Read Section 7.4	Section 7.1: 37, 48, 53 Section 7.2: 13, 17, 21 Section 7.3: 1, 6, 10
16	Review Section 7.4 Read Section 7.7	Section 7.2: 25, 29 Section 7.3: 14, 18, 22 Section 7.4: 2, 3, 5
17	Review Section 7.7 Read Section 9.1	Section 7.3: 27, 28 Section 7.4: 10, 20 Section 7.7: 1, 9, 18
18	Review Section 9.1 Read Section 8.1	Section 7.4: 34, 37 Section 7.7: 21, 30, 34 Section 9.1: 1, 8, 15
19	Review Section 8.1 Read Section 8.2	Section 7.7: 47, 67, 84a Section 9.1: 22, 27, 29, 49,52ab Section 8.1: 1,7, 13
20	Review Section 8.2 Read Section 8.3	Section 8.1: 20, 27,34 Section 8.2: 1,8, 15
21		**Team Project I:**
22	Review Section 8.3 Read Section 8.4	Section 8.1: 40, 41, 43 Section 8.2 : 20, 27,33 Section 8.3: 1, 8, 15
23	Review Section 8.4 Read Section 8.5	Section 8.2: 43, 57, 61 Section 8.3: 22, 29, 36 Section 8.4: 1,5, 9
24	Review Section 8.5 Read Section 8.6	Section 8.3: 43, 49, 57 Section 8.4: 13, 17,21 Section 8.5: 1, 6,11
25	Review Section 8.6 Read Section 8.7	Section 8.4 25, 29, 33 Section 8.5: 16, 21, 26 Section 8.6: 1,7, 13
26	Review Section 8.7 Read Section 8.8	Section 8.5: 31, 44, 45 Section 8.6: 19, 23, 25 Section 8.7: 1, 5
27	Review Section 8.8 Read Section 8.9	Section 8.6: 43, 45 Section 8.7: 13, 15, 19 Section 8.8: 1,6, 9
Assignment	***Reading***	***Exercises***
28	Review Section 8.9 Read Section 9.2	Section 8.7: 33, 36, 38 Section 8.8: 16, 21, 26 Section 8.9: 1, 7, 13
29	Review Section 9.2 Read Section 9.3	Section 8.8: 30, 48 Section 8.9; 20, 26, 28 Section 9.2: 1,7, 10
30	Review Section 9.3	Section 8.9: 41, 43 Section 9.2: 15,22, 26 Section 9.3: 1, 9, 30
31		**Team Project II:**
32	Series Solution of Differential Equations	Section 9:3: 13, 42, 51 Series Solutions: 2, 3, 7

Some Useful Formulas for Homework Assignment 1

$$(e^x)' = e^x$$

$$\int e^x = e^x + C$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Caution!

In general, it is almost always the case that

$$(a+b)^2 \neq a^2 + b^2$$

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

$$\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}$$

MATH 122 : CALCULUS II

Some Problems Related to the Material of This Course

1. Lowerbury College is located on the straight stretch of the shoreline of the Atlantic Ocean. The night before the first calculus exam, there is an electrical power failure at the college. The only available light on campus comes from a lighthouse 3 miles from the shore. To guide ocean-going ships, the light in the lighthouse revolves at a rate of 8 revolutions per minute. Newton Isaac, one of the calculus students, hits upon the idea of walking along the shoreline at a pace that will keep the revolving light on the calculus book he is trying to study. Discuss how Newton must adjust his speed. In particular, find how fast Newton must run at the instant the light makes a 45 degree angle with the shoreline.

2. In a condenser discharging electricity, the rate of change of the voltage in volts per second is proportional to the voltage. In how many seconds will the voltage decrease to ten percent of its original value?

3. Along what curve should a skier stay if, subject to the force of gravity only, she wants to slide from the top of a mountain to the shelter at the base n the shortest time?

4. Newton Isaac purchased a savings bond for $500 with interest compounded continuously at 7% per year. When will the bond be worth $1000?

5. How long is an ellipse?

6. When a person takes 100-mg tablet of a asthma drug orally, the rate R at which the drug enters the bloodstream is predicted to be $R = 5(0.95)^t$ mg/minute. Who long does it take for 50 mg to enter the bloodstream?

7. Find a good approximation for π.

8. Our friend Newton decides to test the theory that if you dig a hole deep enough into the ground, you can reach China. He succeeds in boring a straight tunnel through the center of the earth. Then he falls in. Show that it takes less than an hour to reach the other side of the world.

9. One of Kepler's laws states that each planet moves along an ellipse with the sun at one of the foci and that each planet moves with constant areal velocity. Find the speed of the planet when it is furthest from the sun.

10. Newton walks east at a speed of 4 miles per hour and notices that the cold wind appears to be blowing directly from the north. On doubling his speed, he finds that the wind appears to be blowing from the northeast. What is the velocity of the wind?

11. Where is the most effective place to station a helicopter for fighting fires in a large forest? Assume such things as: the helicopter travels at a constant speed in a straight line toward a fire, that it goes to the fire as soon as the fire starts, that the fire spreads in a circle whose radius is proportional to the duration of the fire, and the forest is flat, with the trees uniformly distributed.

12. The Gateway Arch to the West in St. Louis has the shape of an inverted caternary. Rising 630 feet at its center and stretching 630 feet across its base. Approximate the total open area under the arch and the total length of the arch.

13. How far should you stand from the *Mona Lisa* to get the best view?

14. How much work is done in propelling a space capsule weighing 2000 pounds to a height of 400 miles?

15. Isaac Newton's law of cooling says that the rate of change of temperature of a cooling body is proportional to the difference between the body and its surroundings. Newton Isaac steps out of Warner Hall boiling mad at his calculus teacher. The outside temperature is -10

degrees. If Newton simmers down to 100 degrees in 15 minutes, how long will it be before he freezes to death?

16. The reliability *R(t)* of a product is the probability that it will not require repair for at least *t* years. To design a warranty guarantee, a manufacturer must know the average time of service before first repair of a product. This is given by the improper integral $\int_0^{\square} (-t)R'(t)\,dt$. For many high-quality products, *R(t)* has the form e^{-kt} for some positive constant k . Find an expression in terms of k for the average time of service before repair. Is it possible to manufacture a product for which R(t) = 1/(t+1).

17. One day it started snowing at a heavy and steady rate. A snowplow started out at noon, going 2 miles the first hour and 1 mile the second hour. What time did it start snowing?

18. How much dough does it take to make a doughnut?

19. Suppose 50 pounds of raisins are mixed into 1000 pounds of oatmeal to make oatmeal raisin cookies. If 10,000 cookies are made, how likely is it to find a cookie with more than 3 raisins?

20. Suppose the rate of spread of an infectious disease in a population is jointly proportional to the number of people who are healthy and the number who are sick with the disease. Determine how many people are sick 10 days after the epidemic starts.

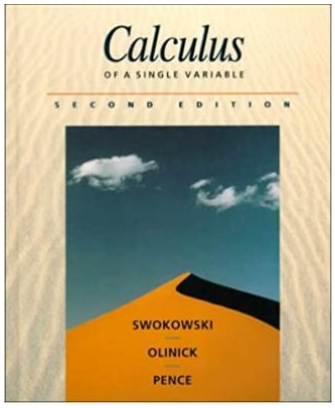
What To Do By Tomorrow

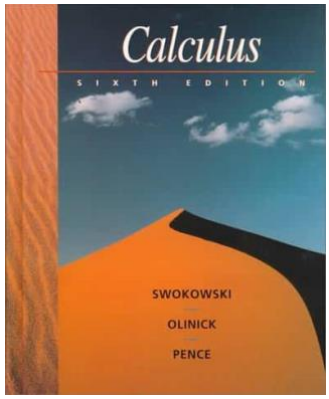
1) If you are still awaiting shipment of our text, you can find two copies on reserve at the Davis Family Library. There are links to the various chapters of the book at our course website:

Skim through the material in Chapters 1 - 4; almost all of it should appear familiar although you may not be able today to remember and apply the formulas the way you could a few months ago! Don't worry; you'll quickly regain your old expertise. You will not need all of this material for the first few assignments either.

Familiarize yourself with the structure of the book: table of contents, precalculus review, various appendices, answers to selected exercises, the index, etc.

Review the first six sections of Chapter 4 with particular attention to Section 4.5: *The Fundamental Theorem of Calculus.*





2) Familiarize yourself with the material in this packet.

3) Read the handout on *Measuring Income Inequality: The Lorenz Curve and the Gini Index.*

4) Purchase a binder to store the various handouts about the course that will be distributed, your class notes and the homework. You will accumulate a large number of loose sheets of paper from this course during the term; it's very helpful to keep them organized. You may also wish to obtain some graph paper.

5) Complete and submit Assignment 0.

ON STUDYING AND LEARNING MATHEMATICS

Past students have found that some ways are far more effective than others in studying and learning mathematics. Here are some suggestions and pointers that may help you in budgeting the time you can devote to mathematics, preparing for examinations, and learning and understanding the material in a way that promotes long-range retention:

1. Do all reading assignments actively. Keep a pencil and scratch paper at hand. Mark up the pages of the book. Write in any questions you may have. Verify examples given by writing out the details yourself.

2. Plan to do all reading assignments several times. In mathematics courses, reading assignments are seldom more than a few pages long. They often contain, however, subtle ideas which require repeated study before they are mastered. You should read the appropriate section of the text before the class in which it will be discussed, read it again before beginning the homework assignment, and read it a third time after you have completed the homework.

3. Follow the advice in (1) above when reviewing your lecture notes. You should try to go over your lecture notes as soon as possible after the class session has ended. Definitely review the notes before attempting the homework.

4. Do all homework sets on time. Don't let yourself fall behind. If you have difficulty with a problem, especially one that is more theoretical, do the following:

   (a) Write out the relevant definitions and results. It may now be a small step to complete the problem.

   (b) Ask whether you can think of a simpler but related problem, and tackle that one first. Is there a special case of the general result? Do you know how to solve the problem in this special case? This approach usually provides insight for attacking the original problem.

5. Do not spend hours sitting still, thinking, reading, studying and reviewing problem solutions! While these approaches may be helpful for other courses and some time should be spent on these activities in mathematics courses, there are more productive paths to learning in mathematics. Spend your time writing out solutions to new problems, deriving relationships, writing down clear

definitions, and outlining the steps of a proof. These activities provide a better way to prepare for an examination.

6. Pay a great deal of attention to definitions. Write them out yourself and think about them. Write out examples that do and do not satisfy the definitions. Ask yourself how the definition says something different from its intended meaning if the order of the words is shifted.

7. Begin reviewing for examinations a week early. Use small chunks of time. Tackle those topics you have found difficult; with hindsight they are often easier. Do NOT plan on spending a whole day of study just before an exam. This is almost always an inefficient way to budget your time.

8. Review solutions for homework problems as soon as you get them, and write up (for your own enlightenment) those problems which caused you difficulty.

9. Write down questions that arise as you go along. Bring them with you to class, to review sessions, and to your instructor's office hours.

ON PROBLEM SOLVING

A major part of your time in Calculus and other courses is devoted to solving problems. It is worth your while to develop sound techniques. Here are a few suggestions.

Think. Before plunging into a problem, take a moment to think. Read the problem again. Think about it. What are its essential features? Have you seen a problem like it before? What techniques are needed?

Try to make a rough estimate of the answer. It will help you understand the problem and will serve as a check against unreasonable answers. A car will not go 1,000 miles in 3 hours; a weight dropped from 10,000 feet will not hit the earth at 5 mph; the volume of a tank is not -275 gal.

Examine the data. Be sure you understand what is given. Translate the data into mathematical language. Whenever possible, make a clear diagram and label it accurately. Place axes to simplify computations. If you get stuck, check that you are using all the data.

Avoid sloppiness.

(a) *Avoid sloppiness in language.* Mathematics is written in English sentences. A typical mathematical sentence is "$y = 4x + 1$." The equal sign is the verb in this sentence; it means "equals" or "is equal to." The equal sign is not to be used in place of "and", nor as a punctuation mark.

Quantities on opposite sides of an equal sign must be equal.

Use short simple sentences. Avoid pronouns such as "it" and "which". Give names and use them. Consider the following example.

"To find the minimum of it, differentiate it and set it equal to zero, then solve it which if you substitute it, it is the minimum."

Better: "To find the minimum of $f(x)$, set its derivative $f'(x)$ equal to zero. Let x_0 be the solution of the resulting equation. Then $f(x_0)$ is the minimum value of $f(x)$."

(b) *Avoid sloppiness in computation.* Do calculations in a sequence of neat, orderly steps. Include all steps except utterly trivial ones. This will help eliminate errors, or at least make errors easier to find. Check any numbers used; be sure that you have not dropped a minus sign or transposed digits.

(c) *Avoid sloppiness in units.* If you start out measuring in feet, all lengths must be in feet, all areas in square feet, and all volumes in cubic feet. Do not mix feet and acres, seconds and years.

(d) *Avoid sloppiness in the answer.* Be sure to answer the question that is asked. If the problem asks for the maximum value of $f(x)$, the answer is not the point where the maximum occurs. If the problem asks for a formula, the answer is not a number.

EXAMPLE 6.1 Find the minimum of $f(x) = x^2 - 2x + 1$.

Solution 1:

$$2x - 2$$
$$x = 1$$
$$1^2 - 2 \cdot 1 + 1$$
$$0$$

Unbearable. This is just a collection of marks on the paper. There is absolutely no indication of what these marks mean or of what they have to do with the problem. When you write, it is your responsibility to inform the reader what you are doing. Assume he is intelligent, but not a mind reader.

Solution 2:

$$\frac{df}{dx} = 2x - 2 = 0 = 2x = 2 = x = 1$$
$$= f(x) = 1^2 - 2 \cdot 1 + 1 = 0.$$

Poor. The equal sign is badly mauled. This solution contains such enlightening statements as "0 = 2 = 1," and it does not explain what the writer is doing.

Solution 3:

$$\frac{df}{dx} = 2x - 2 = 0, \quad 2x = 2, \quad x = 1.$$

This is better than Solution 2, but contains two errors. Error 1: The first statement, "$\frac{df}{dx} = 2x - 2 = 0$," muddles two separate steps. First the derivative is computed, then the derivative is equated to zero. Error 2: The solution is incomplete because it does not give what the problem asks for, the minimum value of *f*. Instead, it gives the point *x* at which the minimum is assumed.

Solution 4: The derivative of *f* is

$$f' = 2x - 2.$$

At a minimum, **f '** = 0. Hence

$$2x - 2 = 0, \quad x = 1.$$

The corresponding value of *f* is

$$f(1) = 1^2 - 2 \cdot 1 + 1 = 0.$$

If $x > 1$, then $f'(x) = 2(x-1) > 0$, so *f* is increasing. If $x < 1$, then f '(x) = 2(x-1) < 0, so *f* is decreasing. Hence f is minimal at x = 1, and the minimum values of f is 0.

This solution is absolutely correct, but long. For homework assignments the following is satisfactory (check with your instructor):

Solution 5:

$$f'(x) = 2x - 2.$$

At min, $f' = 0$, $2x - 2 = 0$, $x = 1$. For $x > 1$, $f'(x) = 2(x-1) > 0$, f□: for $x < 1$, $f'(x) = 2(x-1) < 0$, f□.

Hence $x = 1$ yields min,

$$f_{min} = f(1) = 1^2 - 2 \cdot 1 + 1 = 0.$$

The next solution was submitted by a student who took a moment to think.

Solution 6:

$$f(x) = x^2 - 2x + 1 = (x-1)^2 \geq 0.$$

But

$$f(1) = (1-1)^2 = 0.$$

Hence the minimum value of *f(x)* is 0.

- from *A First Course in Calculus*
by Flanders, Korfhage and Price

Advice on Reading Your Mathematics Textbooks

[There are many excellent essays online with advice on how to read effectively a mathematics book. Here is an adaption of one of them. I haven't been able to track down the original author's name to give full credit.]

Reading the textbook is important for succeeding academically, and this is also true in your mathematics classes. However, reading mathematics is different from other types of reading. Getting the most out of a mathematics textbook will require more than just skimming through the pages. Below are some tips for helping you get the most from your mathematics text.

• **Focus on concepts, not exercises**. The most important material in a mathematics textbook is found in the prose, not in the exercises at the end of the section. In the past, you may have opened your mathematics book only when doing problem sets and exercises (looking at the rest of the book only for examples which mirror the current assigned homework). You must rid yourself of this bad habit now. Instead, set aside time to read the text when you are not working on a homework assignment. This will enable you to truly focus on the mathematical concepts at hand.

There are an infinite number of types of mathematics problems, so there is no way to learn every single problem-solving technique. Mathematics is about ideas. The mathematics problems that you are assigned are expressions of these ideas. If you can learn the key concepts, you will be able to solve any type of problem (including ones you have never seen before) that involves those concepts.

• **Read the text more than once.** You cannot read mathematics in the same way as you would read a newspaper or a novel. Many of the ideas presented in a typical college mathematics course have confounded brilliant minds in centuries past. So it is not unexpected that you may have difficulty learning these same ideas if you quickly scan through the reading assignments only once. You should expect to go through the each reading assignment several times before you can gain a full understanding of the material.

• **When reading through for the first time, look for the big ideas**. The first time you read through a chapter of the textbook, you should be thinking to yourself: "What is the main point of the chapter?" Look for the big picture. The details are important, but you need to be aware of the forest first before focusing on the trees.

• **The second time through, fill in details**. After you get the big picture, you should then look at the details. Take some time to think about each of the definitions, theorems, and formulas you encounter (more on this later).

• **Read with paper and pen**. As you are reading through the text, you should be writing notes and verifying any parts of which you are skeptical. Check any calculations. Rewrite definitions and theorems in your own words.

See if you can come up with your own examples. Ask yourself about special cases of the theorems you read.

• **Read the narrative**. There is a story to be told in mathematics. What is the progression of ideas being told? Don't just skip to the formulas and examples, but instead follow the development of the ideas and concepts presented.

• **Study the examples**. What points do each of the examples illustrate? Some examples are extreme cases. Other examples are supposed to illustrate "typical" situations.

• **Read the pictures**. There are good reasons for the many pictures and graphs in mathematics texts. You should be asking yourself what features of the picture are important to the key concepts. Focus on how each picture illustrates a particular idea.

• **Learn the vocabulary and the language.** Pay attention to definitions and what they mean. Mathematics language is very precise, and a word in a mathematical context may have a different meaning than when it is used in everyday conversation. In mathematics, great care is taken to explicitly and precisely define the notions being considered. In addition, mathematical definitions and language are crafted in such a way to convey sophisticated notions in as simple and concise a manner as possible.

• **Learn the theorems and what they mean.** Theorems are vital bricks to building mathematical knowledge. When you see a theorem in a mathematics text, look at it very closely. What does it say? What are its hypotheses? What implications does it have? Are there special cases you should be aware of ? Can you think of examples to which the theorem applies? Can you think of examples that do not satisfy the hypotheses and the conclusion of the theorem?

• **Use the index and the appendices**. Know what every word means. Make sure that you understand all of the words and ideas. If there is a particular word which you do not know (or which you want to know better), look it up. Use the table of contents or the index to help you.

• **Make a note of things you do not understand, and ask for help afterwards.** Even after following all of the above advice, you might still find some of the ideas confusing. That is to be expected; material such as this is often hard to internalize when one first encounters it. If there is something that you do not understand, make a note of it. Write down any questions you may have. You then can bring up these issues with your instructor or a classmate.