MATH 122 CALCULUS II – Some Optional Extra Credit Problems

Due: Friday, December 5

This is a pledged assignment; write and sign the Honor Code pledge.

Feel free to turn in as many or as few of these problems as you wish.

Do not consult other books, journals, internet resources, ChatGPT, etc or ask anybody outside the class. Feel free to work together with other students in the course, should you find this helpful. If you work with others, write your own version of the solutions, and indicate at the beginning with whom you collaborated.

We need to see complete answers, indicating all the steps you took. If you believe that you have made some progress although you are not done, turn in what you did, which may receive some partial extra credit, depending on what you actually did and how far you got.

Turning something in does not automatically mean you will receive extra credit. There has to be something there that constitutes meaningful progress towards the solution.

Display all the intermediate algebra. Do not skip steps. Asking a calculator or a computer algebra system such as *Maple* or *Mathematica* for the maximum of a function, or for its derivative, antiderivative or anything like that, is skipping steps.

1. The functions g and f are defined by $g(x) = sin^4 x + cos^4 x$ and $f(x) = \frac{1}{g(x)}$.
   (a) What are the maximum and minimum values of these two functions?
   (b) Show that f is continuous for all real values of x.
   (c) Determine an antiderivative for $f(x)$.

2. Evaluate $\int_1^2 \frac{(\ln w)^2}{w^3}\, dw$

3. Find $\int \frac{1}{17+\sin x}\, dx$

4. Find $\lim_{x \to 0^+} \left(e^{-\frac{1}{2x}}\right)^x$

5. Find $\lim_{x\to 0^+}\left(x^{\frac{1}{\ln(3x)}}\right)$

6. Suppose f is a continuous function on the interval [0, 2] and all we know about f is that $f(0) = f(2)$.
Prove that there is at least one number x in $[1,2]$ such that $f(x) = f(x-1)$.

7. Find $\int \frac{1}{x^4+1}\,dx$

8. Determine if the improper integrals $\int_0^\infty \sin(x^2)dx$ and $\int_0^\infty \cos(x^2)dx$ converge or diverge.

9. Determine if the improper integral $\int_0^\infty \sin(x)\sin(x^2)dx$ converges or diverges.

10. Let f be a real-valued function such that f, f' and f'' are all continuous on [0,1]. Consider the series $\sum_{k=1}^{\infty} f\left(\frac{1}{k}\right)$.
(a) Prove that if the series $\sum_{k=1}^{\infty} f\left(\frac{1}{k}\right)$ converges, then $f(0) = 0$ and $f'(0) = 0$.
(b) Prove the converse: if $f(0) = 0$ and $f'(0) = 0$, then the series $\sum_{k=1}^{\infty} f\left(\frac{1}{k}\right)$ converges.

11. Find an antiderivative for $\sqrt{\tan x}$.

12. Evaluate the definite integral $\int_0^{\pi/2} arccos\left(\frac{\cos x}{1+2\cos x}\right)\,dx$.

13. Find the sum of the series $\sum_{k=1}^{\infty}\frac{1}{k^2}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{k}\right)$.

14. Find the Taylor series for $f(x) = \sqrt[5]{x} = x^{1/5}$ centered at $c = 32$ and determine the radius of convergence.