Math 121 - Calculus I Name: Solutions

Exam 1: Practice Exam

Please be sure to neatly **show and explain all of your work** and clearly label your answers. Except for your index card, this exam is a closed-book, closed-notebook exam. Calculators are not allowed.

Please write and sign the Honor Pledge here when you are done:

Signed:

Problem	Points .
1	/12
2	/10
3	/10
4	/8
5	/12
6	/8
Total	/60

1. Please compute the following. For each, show all work and clearly explain your reasoning.

(a) $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-4x+3}$

$$\frac{(x-2)(x-1)}{(x-3)(x-1)} = \begin{cases} \frac{x-2}{x-3} & \text{if } x \neq 1 \\ \text{undef} & \text{if } x = 1 \end{cases}$$

$$= \lim_{x\to 1} \frac{(x-2)(x-1)}{(x-3)(x-1)}$$

$$= \lim_{x\to 1} \frac{x-2}{x-3}$$

replace original function with $\frac{x-2}{x-3}$, which equals the original function everywhere but $x=1$, and apply the useful thm.

$$= \frac{1-2}{1-3} = \boxed{\frac{1}{2}}$$

(b) $\lim_{x\to 2^-} \frac{x(x-3)}{x-2}$ (observe: if we plug in 2, we get $\frac{-2}{0}$. This indicates a vertical asymptote)

In the numerator, as $x \to 2^-$, $x(x-3)$ approaches -2, a negative finite #

In the denominator, as $x \to 2^-$, $x-2$ is a very small negative number, approaching 0.

Thus, as $x\to 2^-$, $\frac{x(x-3)}{x-2}$ approaches $\frac{(\text{neg number})}{(\text{very small neg number})}$, so $\boxed{\lim_{x\to 2^-} \frac{x(x-3)}{x-2} = +\infty.}$

(c) $\lim_{x\to -\infty} \frac{4x-x^2}{x-2}$

highest power of x in denominator

$$\lim_{x\to -\infty} \frac{4x-x^2}{x-2} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to -\infty} \frac{4-x}{1-\frac{2}{x}}$$

positive number.

As $x\to -\infty$, the denominator $\to 1$. The numerator $4-x$ grows positive without bound as $x \to -\infty$. Thus $\boxed{\lim_{x\to -\infty} \frac{4-x}{1-\frac{2}{x}} = +\infty.}$

2. Consider the function
$$f(x) = e^{2\sqrt{x-3}}.$$

(a) What is the domain of $f(x)$? Please explain.

$\sqrt{x-3}$ is defined for all $x \geq 3$.

e^x is defined for all x.

Thus $e^{2\sqrt{x-3}}$ is defined for all $x \geq 3$, (a.k.a. $[-3, \infty)$)

(b) Find a formula for $f^{-1}(x)$, the inverse of $f(x)$. Show all work.

$y = e^{2\sqrt{x-3}}$ $\{$ set $y = f(x)$

$\Rightarrow \ln y = 2\sqrt{x-3}$

$\Rightarrow \frac{1}{2}\ln y = \sqrt{x-3}$

$\Rightarrow \left(\frac{1}{2}\ln y\right)^2 = x - 3$

$\Rightarrow \left(\frac{1}{2}\ln y\right)^2 + 3 = x$ $\}$ Solve for x

$\Rightarrow y = \underbrace{\left(\frac{1}{2}\ln x\right)^2 + 3}_{f^{-1}(x)}$ $\{$ interchange x and y

Alternate/equivalent, using $\frac{1}{2}\ln y = \ln(y^{1/2}) = \ln\sqrt{y}$

So:
$\frac{1}{2}\ln y = \sqrt{x-3}$

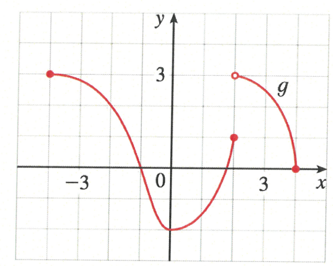
$\Rightarrow \ln\sqrt{y} = \sqrt{x-3}$

$\Rightarrow (\ln\sqrt{y})^2 = x - 3$

$\Rightarrow (\ln\sqrt{y})^2 + 3 = x$

$\Leftrightarrow y = \underbrace{(\ln\sqrt{x})^2 + 3}_{f^{-1}(x)}$

3. Suppose that the following is a graph of $g(x)$.



(a) What is the value of $(g \circ g \circ g)(4)$? Show your work.

$(g \circ g \circ g)(4) = g(g(g(4)))$.

from the graph: $g(g(g(4)))$

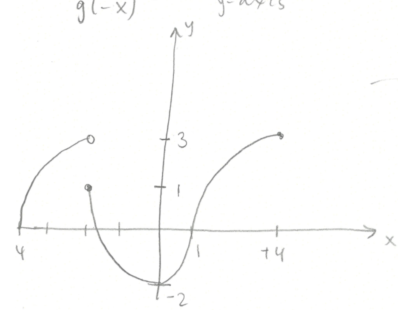
$= g(g(0))$

$= g(-2) = \boxed{2.}$

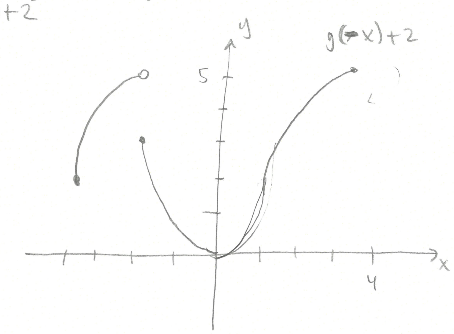
(b) Please sketch a graph of $g(-x) + 2$.

In two stages:

$g(-x)$ ← reflect across y-axis



$g(-x)+2$ ← shift up by 2.



4. (a) Define what it means for a function to be continuous at $x = a$. (What three conditions must hold for the function to be continuous at a?)

f is continuous at a if

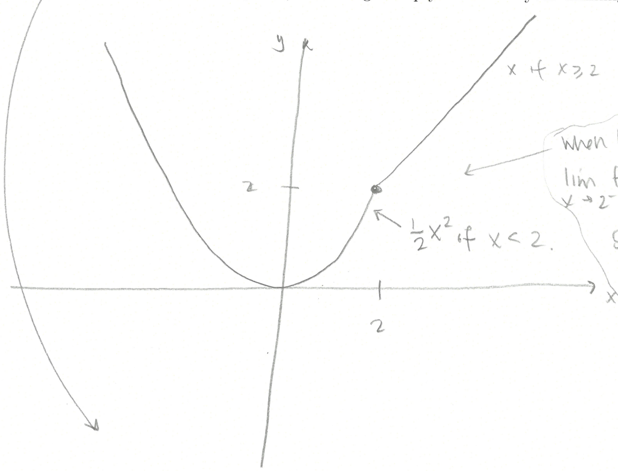
- $f(a)$ exists,
- $\lim_{x\to a} f(x)$ exists
- $\lim_{x\to a} f(x) = f(a)$.

(b) Consider the function given by

$$f(x) = \begin{cases} x & \text{if } x \geq 2 \\ bx^2 & \text{if } x < 2. \end{cases}$$

} The piecewise components of f are cts on their domains. Thus, our only concern is at $x=2$.

For what value of b is this function continuous for all x? Clearly explain your reasoning. (It may help to draw a picture here. It's not required, but it might help you sort out your thinking.)



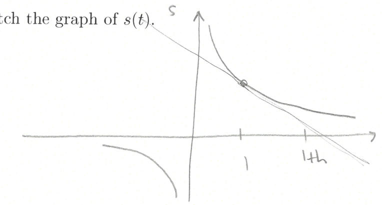
When $b=\frac{1}{2}$,
$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 2 = f$.
So $\lim_{x\to 2} f(x) = 2 = f(2)$,
Thus f is cts.

We need b so that $\lim_{x\to 2^-} bx^2 = 2$. I.e. $b(2)^2 = 2$, so $b = \frac{1}{2}$.
$4b = 2$.

5. Suppose that for $t > 0$, the position of a particle at time t is given by

$$s(t) = \frac{1}{t}.$$

(a) Sketch the graph of $s(t)$.



(b) Find the average velocity of the particle between time $t = 1$ and $t = 1 + h$.

average velocity $= \frac{\Delta s}{\Delta t} = \frac{s(1+h)-s(1)}{(1+h)-1} = \frac{\frac{1}{1+h} - 1}{h}$.

(c) Express the instantaneous velocity of the particle at time $t = 1$ as a limit of average velocities. (Note: *you do not need to compute the limit.*)

instantaneous velocity $= \lim_{h\to 0} \frac{\frac{1}{1+h} - 1}{h}$.

(d) What feature of the graph from part (a) does your answer to part (c) measure?

The instantaneous velocity corresponds to the slope of the tangent line to the graph in (a) at $t=1$.

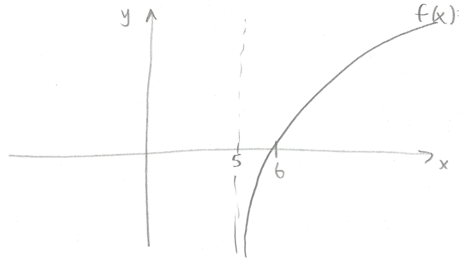
6. Please give an example of a function satisfying each of the following conditions. For each, please give the algebraic expression of your function as well as a graph of your function.

(Hint: your examples do not need to be complicated! Keep in mind our basic functions as you consider this question.)

(These are examples of such functions. Many other examples will work..)

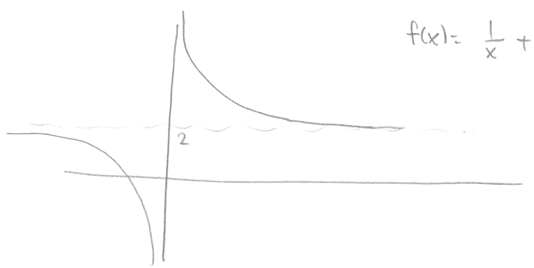
(a) A vertical asymptote at $x = 5$.

$f(x) = \ln(x-5)$



(b) A horizontal asymptote at $y = 2$.

$f(x) = \frac{1}{x} + 2$



(c) A removable discontinuity at $x = 2$.

$f(x) = \frac{x(x-2)}{x-2} = \begin{cases} x, & \text{if } x \neq 2 \\ \text{undef} & \text{if } x = 2. \end{cases}$