Prop Sps $H \leq G$ and $a \in G$. Then

$$aH = Ha \iff aHa^{-1} = H.$$

($\Rightarrow$) Sps $aH = Ha$.

To show $aHa^{-1} = H$, sps $aha^{-1} \in aHa^{-1}$.

Since $aH = Ha$, $ah = h_1 a$ for some $h_1 \in H$. Then

$aha^{-1} = h_1 \in H$, so $aHa^{-1} \subseteq H$.

Now, sps $h_3 \in H$. Then since $aH = Ha$, $ah_4 = h_3 a$

for some $h_4 \in H$. So $h_3 = ah_4 a^{-1} \in aHa^{-1}$,

so $H \subseteq aHa^{-1}$.

We conclude $aHa^{-1} = H$.

($\Leftarrow$) Sps $aHa^{-1} = H$.

To show $aH = Ha$, sps $ah \in aH$....

(the rest is left as an exercise...)