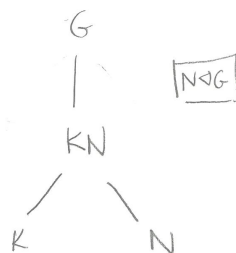


P. 11.2

The Second Isomorphism Theorem states that

$$K/KN \cong KN/N,$$



which implicitly assumes KN/N is a group.

Since $N \triangleleft G$, this is true, as we show here. (In fact, when $N \triangleleft G$, $KN \leq G$, and since $N \triangleleft G$, $N \triangleleft KN$, so KN/N is a group.)

To see that $N \triangleleft G \Rightarrow KN \leq G$, let

$$KN = \{kn \mid k \in K, n \in N\}.$$

- First, since $e \in K$ and $e \in N$, letting $k=e, n=e$, we have $ee=e \in KN$.

- Next, suppose $k, n_1, k_2 n_2 \in KN$. We NTS $k_1 n_1 k_2 n_2 \in KN$, i.e. that it can be expressed as kn for some $k \in K, n \in N$.

But $N \triangleleft G$ so, $Nk_2 = k_2 N$, thus there exists $n_3 \in N$ st.

$$n_1 k_2 = k_2 n_3. \text{ So } k_1 n_1 k_2 n_2 = \underbrace{k_1 k_2}_{\in K} \underbrace{n_3 n_2}_{\in N} \in KN.$$

- Finally, suppose $k, n_1 \in KN$. Then in G , $(kn_1)^{-1} = n_1^{-1} k^{-1}$. We must show that $n_1^{-1} k^{-1}$ can be expressed as kn for some $k \in K, n \in N$.

But again, since $N \triangleleft G$, $Nk_1^{-1} = k_1^{-1}N$ so there exists

$$n_4 \in N \text{ st. } \underbrace{n_1^{-1}}_{\in K} \underbrace{k_1^{-1}}_{\in N} = \underbrace{k_1^{-1}}_{\in K} \underbrace{n_4}_{\in N} \in KN.$$

Thus, under the condition that $N \triangleleft G$, KN is a subgroup of G . Further, $K \leq KN$ and $N \leq KN$

$$\hookrightarrow k \in K = ke \in KN \quad \hookrightarrow ne \in N = ene \in KN$$

and since $N \triangleleft G$, $N \triangleleft KN$ so KN/N is a group.