MATH 122 : CALCULUS II

Some Problems Related to the Material of This Course

1. Lowerbury College is located on the straight stretch of the shoreline of the Atlantic Ocean. The night before the first calculus exam, there is an electrical power failure at the college. The only available light on campus comes from a lighthouse 3 miles from the shore. To guide ocean-going ships, the light in the lighthouse revolves at a rate of 8 revolutions per minute. Newton Isaac, one of the calculus students, hits upon the idea of walking along the shoreline at a pace that will keep the revolving light on the calculus book he is trying to study. Discuss how Newton must adjust his speed. In particular, find how fast Newton must run at the instant the light makes a 45 degree angle with the shoreline.

2. In a condenser discharging electricity, the rate of change of the voltage in volts per second is proportional to the voltage. In how many seconds will the voltage decrease to ten percent of its original value?

3. Along what curve should a skier stay if, subject to the force of gravity only, she wants to slide from the top of a mountain to the shelter at the base n the shortest time?

4. Newton Isaac purchased a savings bond for $500 with interest compounded continuously at 7% per year. When will the bond be worth $1000?

5. How long is an ellipse?

6. When a person takes 100-mg tablet of a asthma drug orally, the rate R at which the drug enters the bloodstream is predicted to be $R = 5(0.95)^t$ mg/minute. Who long does it take for 50 mg to enter the bloodstream?

7. Find a good approximation for π.

8. Our friend Newton decides to test the theory that if you dig a hole deep enough into the ground, you can reach China. He succeeds in boring a straight tunnel through the center of the earth. Then he falls in. Show that it takes less than an hour to reach the other side of the world.

9. One of Kepler's laws states that each planet moves along an ellipse with the sun at one of the foci and that each planet moves with constant areal velocity. Find the speed of the planet when it is furthest from the sun.

10. Newton walks east at a speed of 4 miles per hour and notices that the cold wind appears to be blowing directly from the north. On doubling his speed, he finds that the wind appears to be blowing from the northeast. What is the velocity of the wind?

11. Where is the most effective place to station a helicopter for fighting fires in a large forest? Assume such things as: the helicopter travels at a constant speed in a straight line toward a fire, that it goes to the fire as soon as the fire starts, that the fire spreads in a circle whose

radius is proportional to the duration of the fire, and the forest is flat, with the trees uniformly distributed.

12. The Gateway Arch to the West in St. Louis has the shape of an inverted caternary. Rising 630 feet at its center and stretching 630 feet across its base. Approximate the total open area under the arch and the total length of the arch.

13. How far should you stand from the *Mona Lisa* to get the best view?

14. How much work is done in propelling a space capsule weighing 2000 pounds to a height of 400 miles?

15. Isaac Newton's law of cooling says that the rate of change of temperature of a cooling body is proportional to the difference between the body and its surroundings. Newton Isaac steps out of Warner Hall boiling mad at his calculus teacher. The outside temperature is -10 degrees. If Newton simmers down to 100 degrees in 15 minutes, how long will it be before he freezes to death?

16. The reliability *R(t)* of a product is the probability that it will not require repair for at least *t* years. To design a warranty guarantee, a manufacturer must know the average time of service before first repair of a product. This is given by the improper integral $\int_0^{\square} (-t)R'(t)\,dt$.

    For many high-quality products, *R(t)* has the form e^{-kt} for some positive constant k . Find an expression in terms of k for the average time of service before repair. Is it possible to manufacture a product for which R(t) = 1/(t+1).

17. One day it started snowing at a heavy and steady rate. A snowplow started out at noon, going 2 miles the first hour and 1 mile the second hour. What time did it start snowing?

18. How much dough does it take to make a doughnut?

19. Suppose 50 pounds of raisins are mixed into 1000 pounds of oatmeal to make oatmeal raisin cookies. If 10,000 cookies are made, how likely is it to find a cookie with more than 3 raisins?

20. Suppose the rate of spread of an infectious disease in a population is jointly proportional to the number of people who are healthy and the number who are sick with the disease. Determine how many people are sick 10 days after the epidemic starts.