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ANSWERS

to Selected Exercises

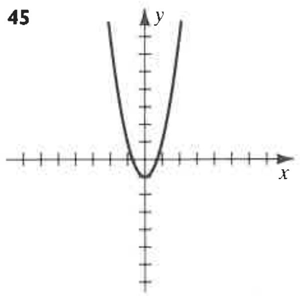
A Student's Solutions Manual to accompany this text is available from your college bookstore. The guide, by Jeffery A. Cole and Gary K. Rockswold, contains detailed solutions to approximately one-third of the exercises as well as strategies for solving other exercises in the text.

Answers are usually not provided for exercises that require lengthy proofs.

PREALGEBRA REVIEW

Exercises A

1 (a) -15 **(b)** -3 **(c)** 11
3 (a) $4-\pi$ **(b)** $4-\pi$ **(c)** $1.5-\sqrt{2}$ **5** $-x-3$
7 $2-x$ **9** $-\frac{6}{5}, \frac{2}{3}$ **11** $-\frac{9}{2}, \frac{3}{4}$ **13** $-2 \pm \sqrt{2}$
15 $\frac{3}{4} \pm \frac{1}{4}\sqrt{41}$ **17** $(12, \infty)$ **19** $[9, 19)$ **21** $(-2, 3)$
23 $(-\infty, -2) \cup (4, \infty)$ **25** $\left(-\infty, -\frac{5}{2}\right] \cup [1, \infty)$
27 $\left(\frac{3}{2}, \frac{7}{3}\right)$ **29** $(-\infty, -1) \cup \left(2, \frac{7}{2}\right]$ **31** $(-3.01, -2.99)$
33 $(-\infty, -2.001] \cup [-1.999, \infty)$
35 $\left(-\frac{9}{2}, -\frac{1}{2}\right)$ **37** $\left[\frac{3}{5}, \frac{9}{5}\right]$
39 (a) The line parallel to the y-axis that intersects the x-axis at $(-2, 0)$
(b) The line parallel to the x-axis that intersects the y-axis at $(0, 3)$
(c) All points to the right of and on the y-axis
(d) All points in quadrants I and III
(e) All points below the x-axis
(f) All points within the rectangle such that $-2 \le x \le 2$ and $-1 \le y \le 1$
41 (a) $\sqrt{29}$ **(b)** $\left(5, -\frac{1}{2}\right)$
43 $d(A, C)^2 = d(A, B)^2 + d(B, C)^2$; area $= 28$

45


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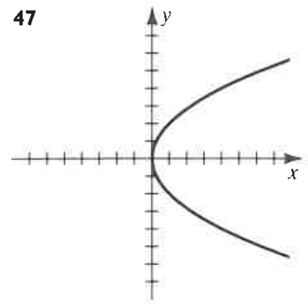

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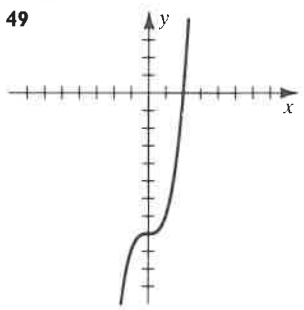

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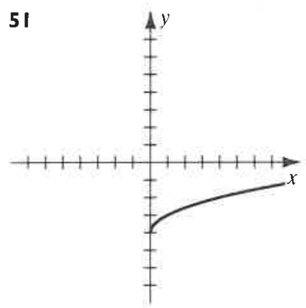

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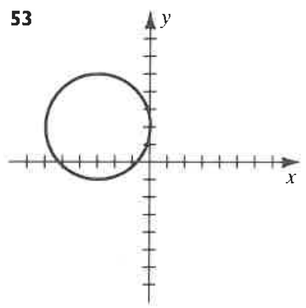

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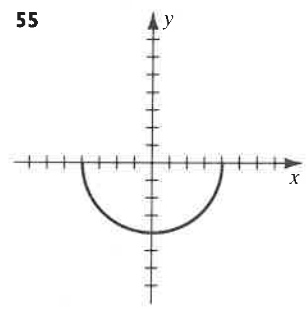
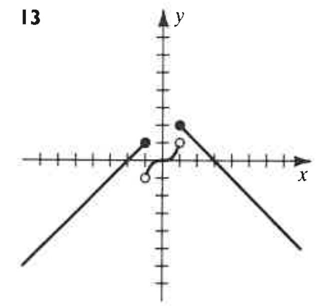


57 $(x-2)^2 + (y+3)^2 = 25$
59 $(x+4)^2 + (y-4)^2 = 16$
61 $4x + y = 17$ **63** $3x - 4y = 12$ **65** $5x - 2y = 18$
67 $-4.04, -0.53$ **69** $0.05, 2.40$ **71** $200 \le m \le 600$
73 $4 \le p < 6$

75 (a) 1 cm **(b)** Capsule: $\frac{11\pi}{96}$ cm^3; tablet: $\frac{\pi}{8}$ cm^3
77 $0 \le v < 30$ **79 (a)** 0 °C **(b)** $\frac{1}{273}$ **(c)** 163.8 °C

Exercises B

1 $-12; -22; -36$
3 (a) $5a-2$ **(b)** $-5a-2$ **(c)** $-5a+2$
(d) $5a+5h-2$ **(e)** $5a+5h-4$ **(f)** 5
5 (a) a^2-a+3
(b) a^2+a+3
(c) $-a^2+a-3$
(d) $a^2+2ah+h^2-a-h+3$
(e) $a^2+h^2-a-h+6$
(f) $2a+h-1$
7 All real numbers except -2, 0, and 2
9 $\left[\frac{3}{2}, 4\right) \cup (4, \infty)$
11 (a) Odd **(b)** Even **(c)** Neither

13


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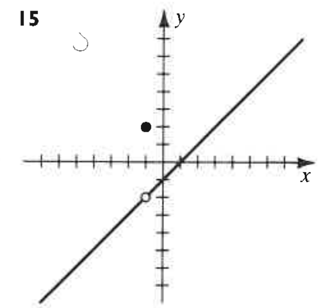

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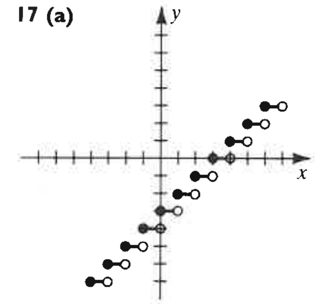

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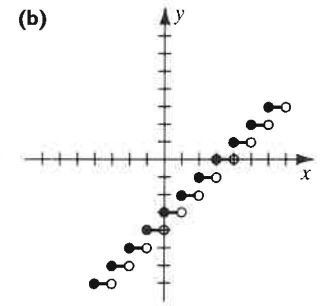

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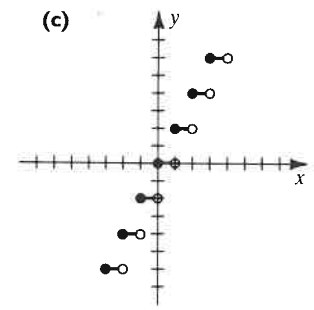

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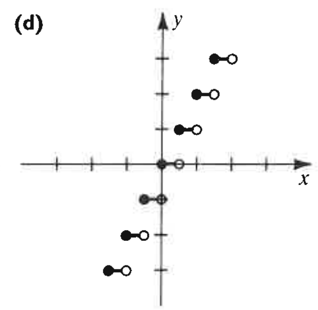

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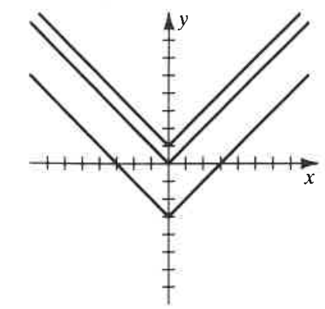

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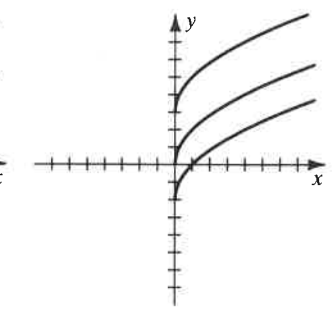

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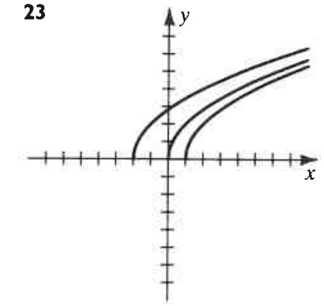

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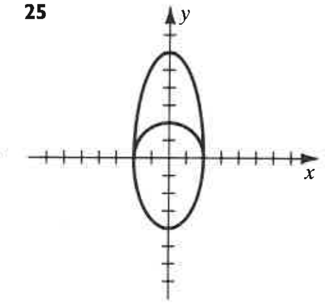

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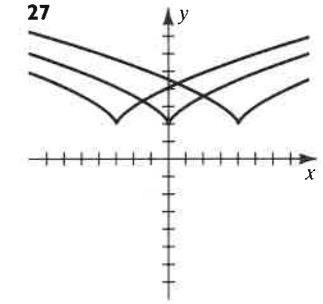
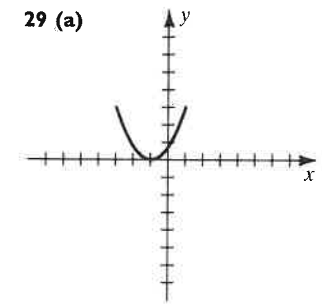


29 (a)


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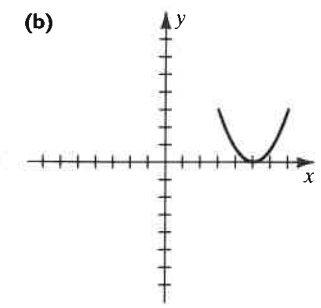

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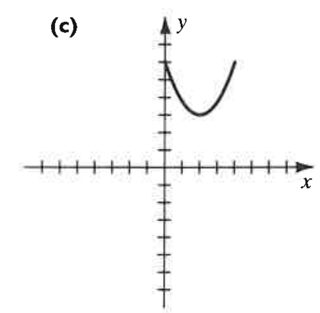

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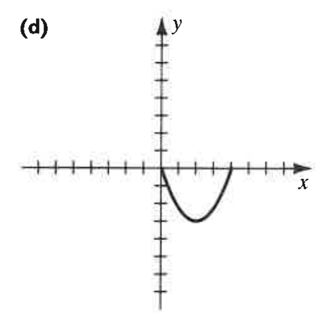

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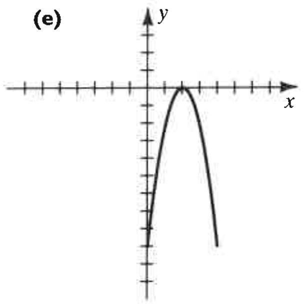

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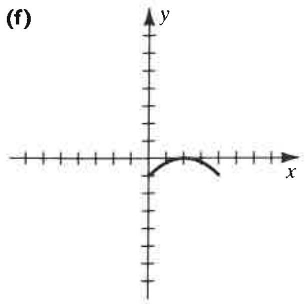

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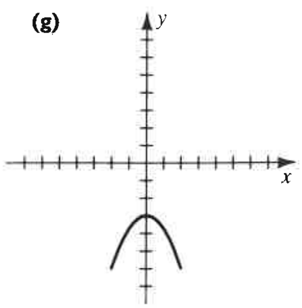

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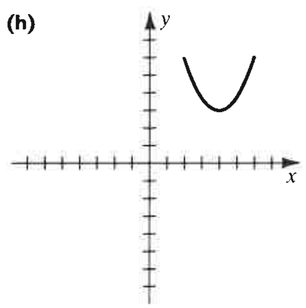


41 (a) $2\sqrt{x+5}$; 0; $x+5$; 1 **(b)** $[-5, \infty)$; $(-5, \infty)$

43 (a) $\dfrac{3x^2+6x}{(x-4)(x+5)}$; $\dfrac{x^2+14x}{(x-4)(x+5)}$; $\dfrac{2x^2}{(x-4)(x+5)}$; $\dfrac{2x+10}{x-4}$

(b) All real numbers except -5 and 4; all real numbers except -5, 0, and 4

45 (a) $x+2-3\sqrt{x+2}$; $[-2, \infty)$

(b) $\sqrt{x^2-3x+2}$; $(-\infty, 1] \cup [2, \infty)$

47 (a) $\sqrt{\sqrt{x+5}-2}$; $[-1, \infty)$

(b) $\sqrt{\sqrt{x-2}+5}$; $[2, \infty)$

49 (a) $\sqrt{28-x}$; $[3, 28]$

(b) $\sqrt{\sqrt{25-x^2}-3}$; $[-4, 4]$

51 (a) $\dfrac{1}{x+3}$; all real numbers except -3 and 0

(b) $\dfrac{6x+4}{x}$; all real numbers except $-\dfrac{2}{3}$ and 0

Exer. 53–60: Answers are not unique.

53 $u = x^2 + 3x$, $y = u^{1/3}$ **55** $u = x - 3$, $y = \dfrac{1}{u^4}$

57 $u = x^4 - 2x^2 + 5$, $y = u^5$

59 $u = \sqrt{x+4}$, $y = \dfrac{u-2}{u+2}$

61 7.91; 5.05 **63** $V = 4x^3 - 100x^2 + 600x$

65 $d = 2\sqrt{t^2 + 2500}$

67 (a) $y = \sqrt{h^2 + 2hr}$ **(b)** 1280.6 mi

69 $d = \sqrt{90{,}400 + x^2}$

71 (a) $y = \dfrac{bh}{a-b}$ **(b)** $V = \dfrac{\pi}{3}h(a^2 + ab + b^2)$

(c) $\dfrac{200}{7\pi}$ ft

Exercises C

1 (a) $\dfrac{5\pi}{6}$ **(b)** $\dfrac{2\pi}{3}$ **(c)** $\dfrac{5\pi}{2}$ **(d)** $-\dfrac{\pi}{3}$

3 (a) 120° **(b)** 150° **(c)** 135° **(d)** −630°

5 $\dfrac{20\pi}{9}$; $\dfrac{80\pi}{9}$ **7** $x = 8$, $y = 4\sqrt{3}$

Exer. 9–16: Answers are in the order sin, cos, tan, cot, sec, csc.

9 $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}, \frac{4}{3}, \frac{5}{4}, \frac{5}{3}$ **11** $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}, \frac{12}{5}, \frac{13}{12}, \frac{13}{5}$

13 $-\frac{3}{5}, \frac{4}{5}, -\frac{3}{4}, -\frac{4}{3}, \frac{5}{4}, -\frac{5}{3}$

Exer. 15–20: Answers are not unique.

15 $4 \cos \theta$ **17** $\sin \theta$ **19** $\sin \theta$

21 (a) $\dfrac{\sqrt{3}}{2}$ **(b)** $\dfrac{\sqrt{2}}{2}$ **23 (a)** $-\dfrac{\sqrt{3}}{3}$ **(b)** $-\sqrt{3}$

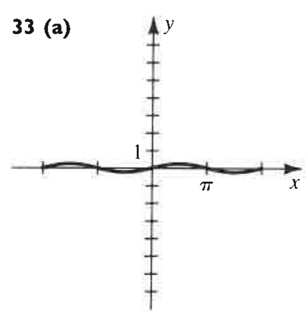
25 (a) -2 **(b)** $\dfrac{2}{\sqrt{3}}$

27 (a) 0.9205 **(b)** 2.3662

29 (a) 0.9781 **(b)** 1.2868

31 (a) −0.8560 **(b)** −0.2958

33 (a)



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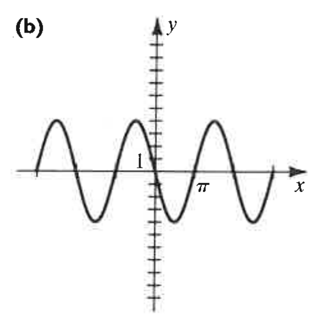
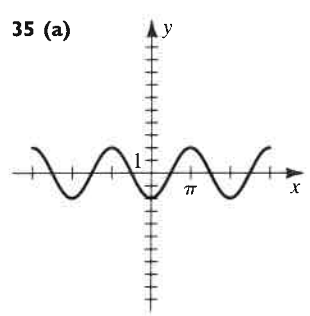


35 (a)



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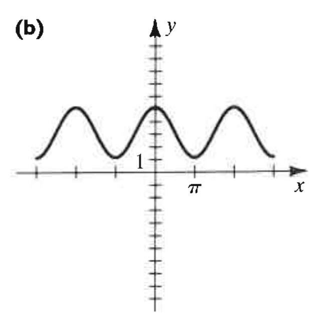
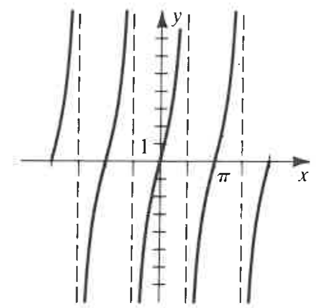


37 (a)



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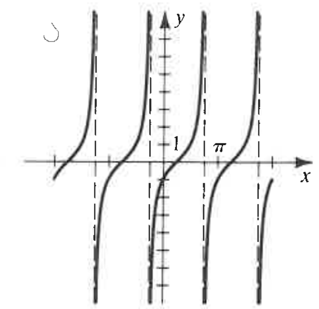


Exer. 39–42: Answers are not unique.

39 $u = \tan^2 x + 4$, $y = \sqrt{u}$ **41** $u = x + \dfrac{\pi}{4}$, $y = \sec u$

43
$$\frac{f(x+h)-f(x)}{h} = \frac{\cos(x+h) - \cos x}{h} = \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} = \frac{\cos x \cos h - \cos x}{h} - \frac{\sin x \sin h}{h} = \cos x\left(\frac{\cos h - 1}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)$$

Exer. 45–54: Typical verifications are given.

45 $(1 - \sin^2 t)(1 + \tan^2 t) = (\cos^2 t)(\sec^2 t) = (\cos^2 t)(1/\cos^2 t) = 1$

47
$$\frac{\csc^2\theta}{1+\tan^2\theta} = \frac{\csc^2\theta}{\sec^2\theta} = \frac{1/\sin^2\theta}{1/\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \cot^2\theta$$

49
$$\frac{1+\csc\beta}{\sec\beta} - \cot\beta = \frac{1}{\sec\beta} + \frac{\csc\beta}{\sec\beta} - \cot\beta = \cos\beta + \frac{\cos\beta}{\sin\beta} - \cot\beta = \cos\beta$$

51
$$\begin{aligned}\sin 3u &= \sin(2u + u) = \sin 2u \cos u + \cos 2u \sin u \\ &= (2 \sin u \cos u) \cos u + (1 - 2 \sin^2 u) \sin u \\ &= 2 \sin u \cos^2 u + \sin u - 2 \sin^3 u \\ &= 2 \sin u(1 - \sin^2 u) + \sin u - 2 \sin^3 u \\ &= 2 \sin u - 2 \sin^3 u + \sin u - 2 \sin^3 u \\ &= 3 \sin u - 4 \sin^3 u = \sin u(3 - 4 \sin^2 u)\end{aligned}$$

53
$$\begin{aligned}\cos^4\frac{\theta}{2} &= \left(\cos^2\frac{\theta}{2}\right)^2 = \left(\frac{1+\cos\theta}{2}\right)^2 \\ &= \frac{1 + 2\cos\theta + \cos^2\theta}{4} \\ &= \frac{1}{4} + \frac{1}{2}\cos\theta + \frac{1}{4}\left(\frac{1+\cos 2\theta}{2}\right) \\ &= \frac{1}{4} + \frac{1}{2}\cos\theta + \frac{1}{8} + \frac{1}{8}\cos 2\theta \\ &= \frac{3}{8} + \frac{1}{2}\cos\theta + \frac{1}{8}\cos 2\theta\end{aligned}$$

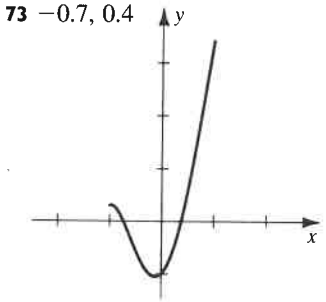
55 $\dfrac{\pi}{12} + \pi n$, $\dfrac{11\pi}{12} + \pi n$, where n denotes any integer

57 $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$ **59** $\dfrac{\pi}{4}, \dfrac{5\pi}{4}$ **61** $0, \pi, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ **63** $0, \pi$

65 3.7408, 5.6840 **67** 1.2275, 4.3691 **69** 2.6816, 3.6016

71 The graph of f appears to pass through the point $(\pi, -1)$.

73 −0.7, 0.4



75 $h = \dfrac{10}{\cot 0.17 - \cot 1.2} \approx 1.84$ km

Exercises D

7 5 **9** −1, 3 **11** 6 **13** $\dfrac{18}{5}$

15 (a) 900 **(b)** 590 **(c)** 349

17 (a) 1.15 mg **(b)** 30% **19** $3 = \log_5 125$

21 $x = \log_3(7 + t)$ **23** $t = \log_{0.7}(2/3)$ **25** $2^5 = 32$

27 $10^3 = 1000$ **29** $7^{5x+3} = m$ **31** $t = 5 \log_a(5/2)$

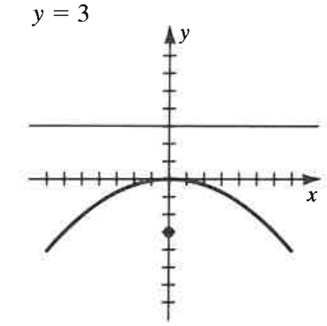
33 $t = \dfrac{1}{C}\log_a\dfrac{A-D}{B}$ **35** 0 **37** Not possible **39** 8

41 4 **43** −2, −1 **51 (a)** 10 **(b)** 30

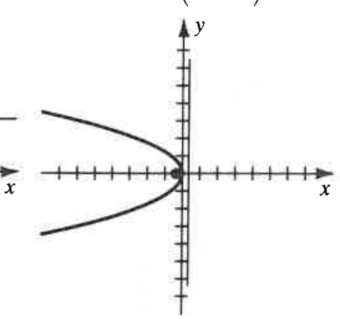
53 (a) 253 million; 271.36 million **(b)** 2089

Exercises E

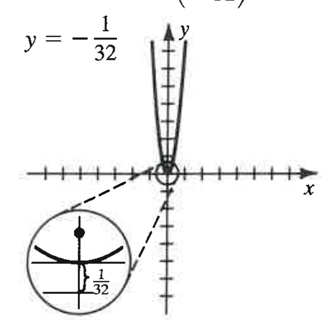
1 $V(0, 0)$; $F(0, -3)$; $y = 3$



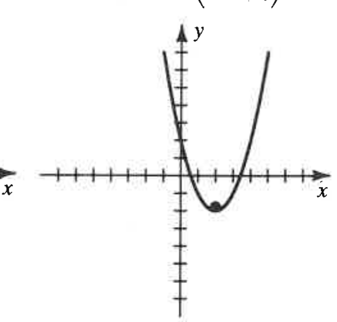
3 $V(0, 0)$; $F\left(-\frac{3}{8}, 0\right)$; $x = \frac{3}{8}$



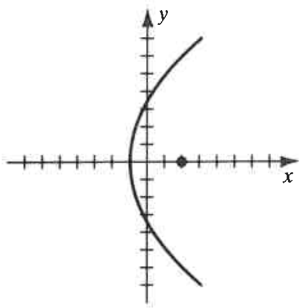
5 $V(0, 0)$; $F\left(0, \frac{1}{32}\right)$; $y = -\frac{1}{32}$



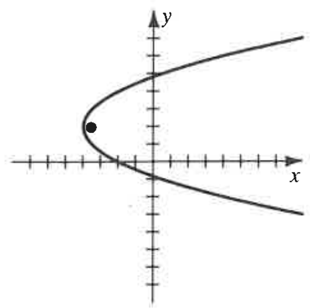
7 $V(2, -2)$; $F\left(2, -\frac{7}{4}\right)$



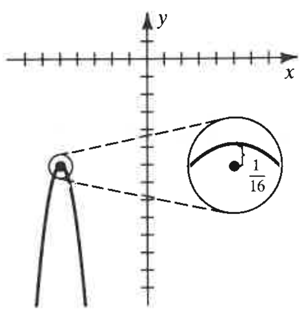
9 $V(-1, 0); F(2, 0)$



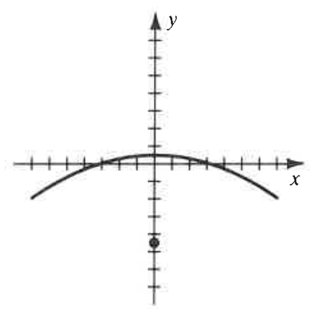
11 $V(-4, 2); F\left(-\frac{7}{2}, 2\right)$



13 $V(-5, -6); F\left(-5, -\frac{97}{16}\right)$



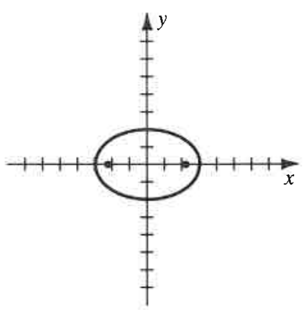
15 $V\left(0, \frac{1}{2}\right); F\left(0, -\frac{9}{2}\right)$



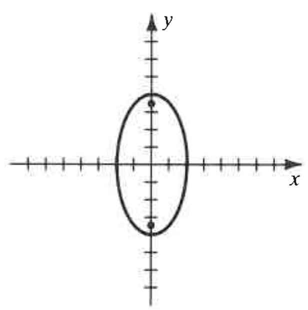
17 $y^2 = 8x$ **19** $(y + 5)^2 = 4(x - 3)$

21 $y^2 = -12(x + 1)$ **23** $3x^2 = -4y$

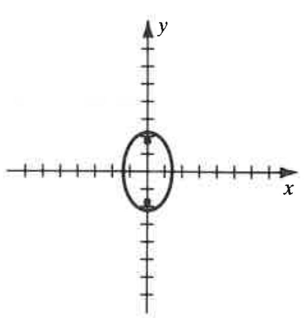
25 $V(\pm 3, 0); F(\pm\sqrt{5}, 0)$



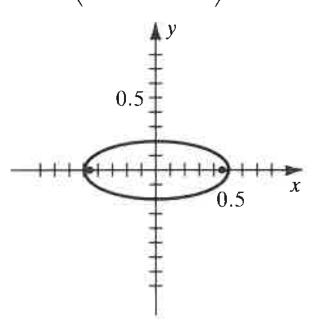
27 $V(0, \pm 4); F(0, \pm 2\sqrt{3})$



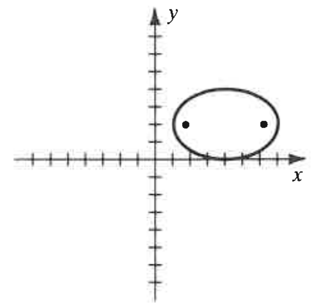
29 $V(0, \pm\sqrt{5}); F(0, \pm\sqrt{3})$



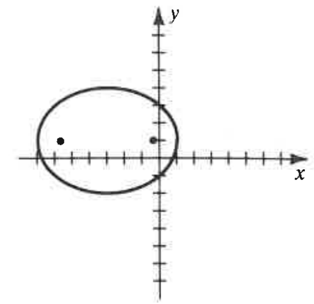
31 $V\left(\pm\frac{1}{2}, 0\right)$; $F\left(\pm\frac{1}{10}\sqrt{21}, 0\right)$



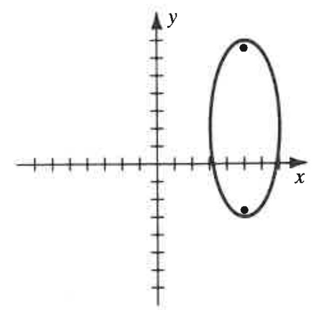
33 $V(4 \pm 3, 2); F(4 \pm \sqrt{5}, 2)$



35 $V(-3 \pm 4, 1)$; $F(-3 \pm \sqrt{7}, 1)$



37 $V(5, 2 \pm 5); F(5, 2 \pm \sqrt{21})$



39 $\frac{x^2}{64} + \frac{y^2}{39} = 1$

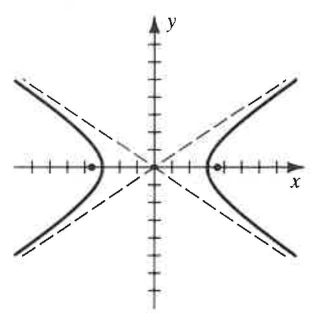
41 $\frac{4x^2}{9} + \frac{y^2}{25} = 1$

43 $\frac{8x^2}{81} + \frac{y^2}{36} = 1$

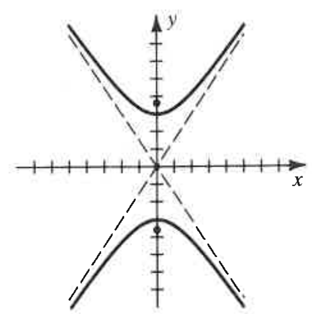
45 $\frac{x^2}{7} + \frac{y^2}{16} = 1$

47 $\frac{x^2}{4} + 9y^2 = 1$

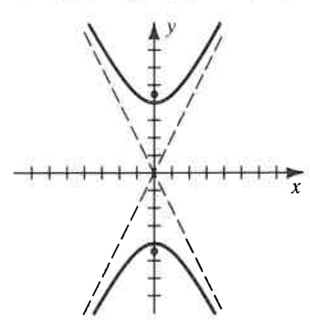
49 $V(\pm 3, 0); F(\pm\sqrt{13}, 0)$



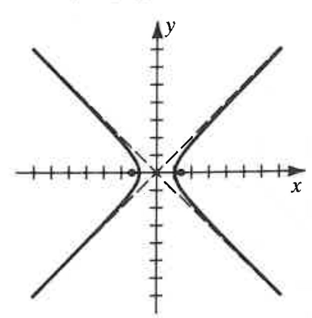
51 $V(0, \pm 3); F(0, \pm\sqrt{13})$



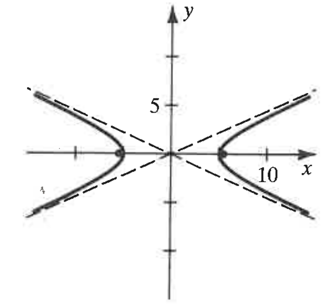
53 $V(0, \pm 4); F(0, \pm 2\sqrt{5})$



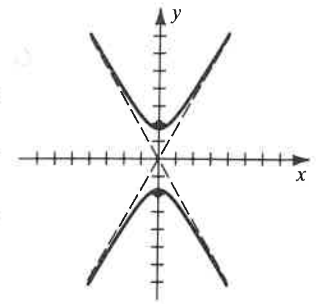
55 $V(\pm 1, 0); F(\pm\sqrt{2}, 0)$



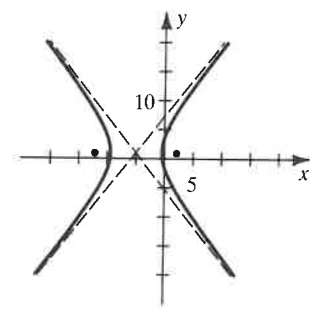
57 $V(\pm 5, 0); F(\pm\sqrt{30}, 0)$



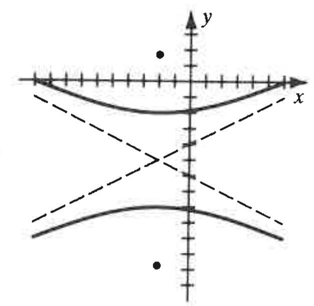
59 $V(0, \pm\sqrt{3}); F(0, \pm 2)$



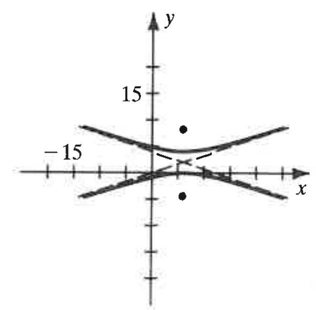
61 $V(-5 \pm 2\sqrt{5}, 1)$; $F\left(-5 \pm \frac{1}{2}\sqrt{205}, 1\right)$



63 $V(-2, -5 \pm 3)$; $F(-2, -5 \pm 3\sqrt{5})$



65 $V(6, 2 \pm 2)$; $F(6, 2 \pm 2\sqrt{10})$



67 $y^2 - \frac{x^2}{15} = 1$

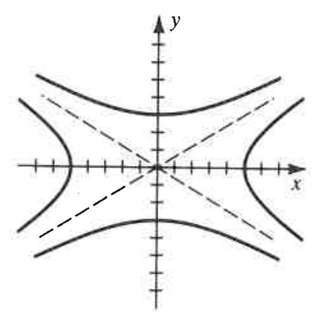
69 $\frac{x^2}{9} - \frac{y^2}{16} = 1$

71 $\frac{y^2}{21} - \frac{x^2}{4} = 1$

73 $\frac{x^2}{9} - \frac{y^2}{36} = 1$

75 $\frac{x^2}{25} - \frac{y^2}{100} = 1$

77 The graphs have the same asymptotes.



79 $\sqrt{84} \approx 9.165$ ft

81 $x = \sqrt{9 + 4y^2}$

CHAPTER ▪ 1

Exercises 1.1

DNE denotes Does Not Exist.

1 -7 **3** 4 **5** 7 **7** π **9** -3 **11** $\frac{7}{2}$

13 4 **15** $\frac{1}{9}$ **17** 32 **19** $2x$ **21** 12 **23** DNE

25 (a) -1 **(b)** 1 **(c)** DNE

27 (a) DNE **(b)** -6 **(c)** DNE

29 (a) DNE **(b)** DNE **(c)** DNE

31 (a) 3 **(b)** 1 **(c)** DNE **(d)** 2 **(e)** 2 **(f)** 2

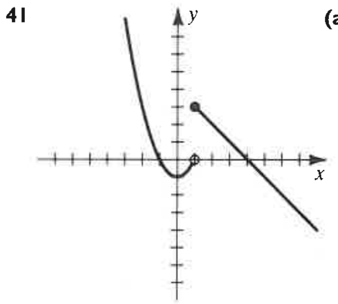
33 (a) 1 **(b)** 1 **(c)** 1 **(d)** 3 **(e)** 3 **(f)** 3

35 (a) 1 **(b)** 0 **(c)** DNE **(d)** 1 **(e)** 0 **(f)** DNE

37 (a) DNE **(b)** DNE **(c)** DNE **(d)** DNE **(e)** 0 **(f)** DNE

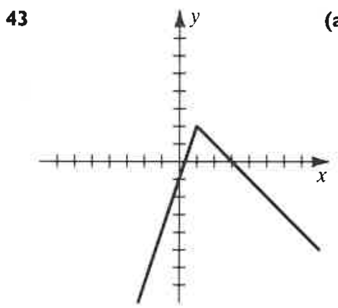
39 (a) -1 **(b)** -1 **(c)** -1 **(d)** DNE **(e)** 1 **(f)** DNE

41



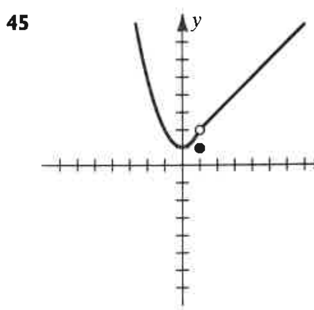
(a) 0 **(b)** 3 **(c)** DNE

43



(a) 2 **(b)** 2 **(c)** 2

45



(a) 2 **(b)** 2 **(c)** 2

47 (a) $T(x) = \begin{cases} 0.15x & \text{if } x \le 20{,}000 \\ 0.20x - 1000 & \text{if } x > 20{,}000 \end{cases}$

(b) \$3000; \$3000

49 (a) $S(x) = \begin{cases} 4 & \text{if } 0 < x \le 10 \\ 4 + 0.4[\![x - 9]\!] & \text{if } x > [\![x]\!] \text{ and } x > 10 \\ 4 + 0.4(x - 10) & \text{if } x = [\![x]\!] \text{ and } x > 10 \end{cases}$

(b) $0.4a$; $0.4(a + 1)$

51 (a) $2g$'s, the g-force at liftoff
(b) Left-hand limit of 8—the g-force just before the second booster is released; right-hand limit of 1—the g-force just after the second booster is released
(c) Left-hand limit of 3—the g-force just before the engines are shut down; right-hand limit of 0—the g-force just after the engines are shut down

Exer. 53–60: A calculator cannot *prove* results on limits. It can only suggest that certain limits exist.

53 (a)

x	$(1 + x)^{1/x}$
−0.02	2.745973
−0.0002	2.718554
−0.000002	2.718285
0.000002	2.718279
0.0002	2.718010
0.02	2.691588

55 (a)

x	$(3^x - 9)/(x - 2)$
1.99	9.833396
1.9999	9.886967
1.999999	9.887505
2.000001	9.887516
2.0001	9.888054
2.01	9.942023

57 (a)

x	$\left(\dfrac{4^{\lvert x\rvert} + 9^{\lvert x\rvert}}{2}\right)^{1/\lvert x\rvert}$
−0.1	6.049510
−0.01	6.004934
−0.001	6.000493
0.001	6.000493
0.01	6.004934
0.1	6.049510

59 (a)

x	$\dfrac{\sin x - 7x}{x \cos x}$
−0.3	−6.296140
−0.03	−6.002851
−0.003	−6.000029
0.004	−6.000051
0.04	−6.005070
0.4	−6.542948

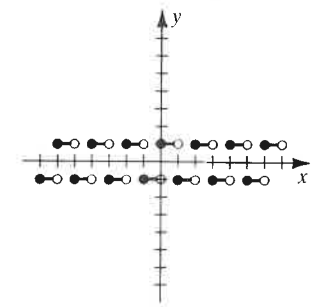
61 (a) Approximate values: 1.0000, 1.0000, 1.0000; −1.2802, 0.6290, −0.8913
(b) The limit does not exist.

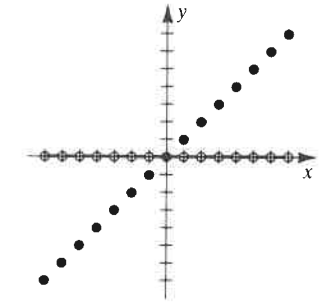
Exercises 1.2

1 (a) $\lim_{t\to c} v(t) = K$ means that for every $\epsilon > 0$, there is a $\delta > 0$ such that if $0 < |t - c| < \delta$, then $|v(t) - K| < \epsilon$.
(b) $\lim_{t\to c} v(t) = K$ means that for every $\epsilon > 0$, there is a $\delta > 0$ such that if t is in the open interval $(c - \delta, c + \delta)$ and $t \ne c$, then $v(t)$ is in the open interval $(K - \epsilon, K + \epsilon)$.
3 (a) $\lim_{x\to p^-} g(x) = C$ means that for every $\epsilon > 0$, there is a $\delta > 0$ such that if $p - \delta < x < p$, then $|g(x) - C| < \epsilon$.
(b) $\lim_{x\to p^-} g(x) = C$ means that for every $\epsilon > 0$, there is a $\delta > 0$ such that if x is in the open interval $(p - \delta, p)$, then $g(x)$ is in the open interval $(C - \epsilon, C + \epsilon)$.
5 (a) $\lim_{z\to t^+} f(z) = N$ means that for every $\epsilon > 0$, there is a $\delta > 0$ such that if $t < z < t + \delta$, then $|f(z) - N| < \epsilon$.
(b) $\lim_{z\to t^+} f(z) = N$ means that for every $\epsilon > 0$, there is a $\delta > 0$ such that if z is in the open interval $(t, t + \delta)$, then $f(z)$ is in the open interval $(N - \epsilon, N + \epsilon)$.
7 0.005 **9** $\sqrt{16.1} - 4$ **11** $|(3.9)^2 - 16| = 0.79$
13 Approximately 0.02396
15 Given any ϵ, choose $\delta \le \epsilon/5$.
17 Given any ϵ, choose $\delta \le \epsilon/2$.
19 Given any ϵ, choose $\delta \le \epsilon/9$.
21 Given any ϵ, let δ be any positive number.
23 Given any ϵ, let δ be any positive number.
31 Every interval $(3 - \delta, 3 + \delta)$ contains numbers for which the quotient equals 1 and other numbers for which the quotient equals −1.
33 Every interval $(-1 - \delta, -1 + \delta)$ contains numbers for which the quotient equals 3 and other numbers for which the quotient equals −3.
35 $1/x^2$ can be made as large as desired by choosing x sufficiently close to 0.
37 $1/(x + 5)$ can be made as large (positively or negatively) as desired by choosing x sufficiently close to −5.
39 *Hint:* Use Theorem (1.3).
41 There are many examples; one is $f(x) = (x^2 - 1)/(x - 1)$ if $x \ne 1$ and $f(1) = 3$.
43 Every interval $(a - \delta, a + \delta)$ contains numbers such that $f(x) = 0$ and other numbers such that $f(x) = 1$.

Exercises 1.3

1 15 **3** −2 **5** 8 **7** $\frac{7}{5}$ **9** 81 **11** 0
13 −13 **15** $5\sqrt{2} - 20$ **17** $\pi - 3.1416$ **19** −23
21 −7 **23** DNE **25** $-\frac{3}{8}$ **27** $-\frac{1}{4}$ **29** 2
31 $\frac{72}{7}$ **33** −2 **35** −2 **37** $-\frac{1}{8}$ **39** $\frac{3}{5}$
41 −810 **43** 3 **45** 1 **47** $\frac{1}{8}$
49 (a) 0 **(b)** DNE **(c)** DNE
51 (a) 0 **(b)** 0 **(c)** 0
53 $(-1)^{n-1}$; $(-1)^n$ **55** 0; 0





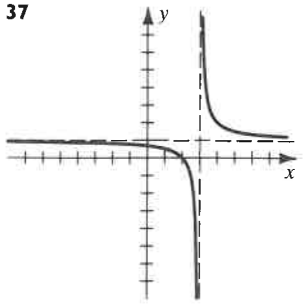
57 (a) $n - 1$ **(b)** n **59 (a)** n **(b)** $n + 1$
65 *Hint:* Let $g(x) = cx^2$.
67 Because Theorem (1.8) is applicable only when the individual limits exist, and $\lim_{x\to 0} \sin \frac{1}{x}$ does not exist
69 (a) 0
(b) If $T < -273\,°\text{C}$, the volume V is negative, an absurdity.
71 (a) DNE **(b)** The image is moving farther to the right.

Exercises 1.4

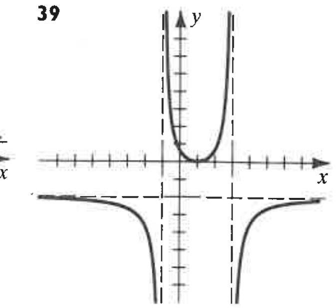
1 (a) $-\infty$ **(b)** ∞ **(c)** DNE
3 (a) $-\infty$ **(b)** ∞ **(c)** DNE
5 (a) $-\infty$ **(b)** $-\infty$ **(c)** $-\infty$
7 (a) ∞ **(b)** $-\infty$ **(c)** DNE
9 (a) ∞ **(b)** ∞ **(c)** ∞ **11** $\frac{5}{2}$ **13** $-\frac{7}{3}$
15 0 **17** $-\infty$ **19** ∞ **21** 1 **23** DNE
25 0.996664442, 0.999966666, 0.999999666, 0.999999996; the limit appears to be 1.
27 $x = -2$, $x = 2$; $y = 0$ **29** None; $y = 2$
31 $x = -3$, $x = 0$, $x = 2$; $y = 0$
33 $x = -3$, $x = 1$; $y = 1$ **35** $x = 4$; $y = 0$

Exer. 37–40: Answers are not unique.

37



39



41 (a) $V(t) = 50 + 5t$; $A(t) = 0.5t$ **(b)** $(t) = t/(10t + 100)$
(c) $c(t)$ approaches 0.1.

Exercises 1.5

1 Jump **3** Removable **5** Jump **7** Infinite **9** Removable
11 Jump **13** Removable **15** Removable **17** Removable
19 $\lim_{x\to 4} f(x) = 12 + \sqrt{3} = f(4)$
21 $\lim_{x\to -2} f(x) = 19 - \frac{1}{\sqrt{2}} = f(-2)$
23 f is not defined at −2. **25** $\lim_{x\to 3} f(x) = 6 \ne 4 = f(3)$
27 $\lim_{x\to 3} f(x) = 1 \ne 0 = f(3)$ **29** $\lim_{x\to 0} f(x) = 1 \ne 0 = f(0)$
31 −3, 2 **33** −2, 1
35 If $4 < c < 8$, $\lim_{x\to c} f(x) = \sqrt{c - 4} = f(c)$.
$\lim_{x\to 4^+} f(x) = 0 = f(4)$ and $\lim_{x\to 8^-} f(x) = 2 = f(8)$
37 If $c > 0$, $\lim_{x\to c} f(x) = \frac{1}{c^2} = f(c)$. **39** $\left\{x\colon x \ne -1, \frac{3}{2}\right\}$
41 $\left[\frac{3}{2}, \infty\right)$ **43** $(-\infty, -1) \cup (1, \infty)$ **45** $\{x\colon x \ne -9\}$
47 $\{x\colon x \ne 0, 1\}$ **49** $[-5, -3] \cup [3, 4) \cup (4, 5]$
51 $\left\{x\colon x \ne \frac{\pi}{4} + \frac{\pi}{2}n\right\}$ **53** $\{x\colon x \ne 2\pi n\}$
55 $\frac{5}{2}$ **57** $c = d = 8$ **59** $c = \sqrt[3]{w - 1}$
61 $c = \frac{1}{2} + \frac{1}{2}\sqrt{4w + 1}$
63 $f(0) = -9 < 100$ and $f(10) = 561 > 100$. Since f is continuous on $[0, 10]$, there is at least one number a in $[0, 10]$ such that $f(a) = 100$.
65 $h(3) = -12 < 0$ and $h(4) = 58 > 0$. Since h is continuous on $[3, 4]$, there is at least one number a in $[3, 4]$ such that $h(a) = 0$.
67 $g(35°) \approx 9.79745 < 9.8$ and $g(40°) \approx 9.80180 > 9.8$. Since g is continuous on $[35°, 40°]$, there is at least one latitude θ between 35° and 40° such that $g(\theta) = 9.8$.
69 −1.341 **71** −0.921, −0.154, 0.936, 1.888
73 ±12.141 **75** $x \approx 5.586, 8.414$ **77** $x \approx -1.521$

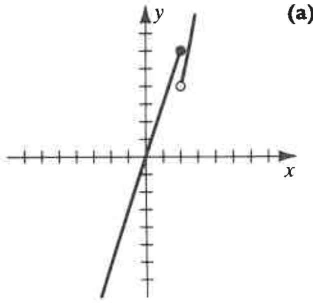
Chapter 1 Review Exercises

1 13 **3** $-4-\sqrt{14}$ **5** $\frac{7}{8}$ **7** $\frac{32}{3}$ **9** ∞

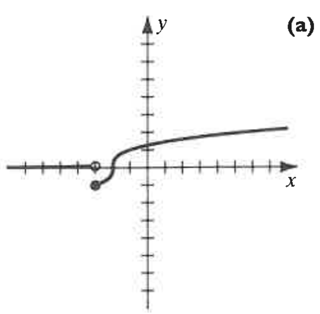
11 3 **13** -1 **15** $4a^3$ **17** $\frac{1}{3}$ **19** $\frac{3}{2}$

21 0 **23** $-\infty$ **25** $-\infty$

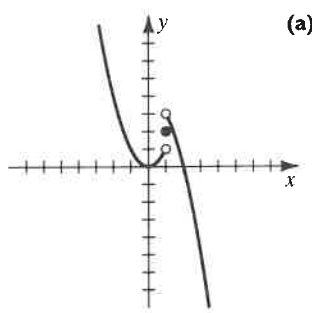
27 **(a)** 6 **(b)** 4 **(c)** DNE



29 **(a)** $\frac{1}{11}$ **(b)** -1 **(c)** DNE



31 **(a)** 1 **(b)** 3 **(c)** DNE



33 Given any ϵ, choose $\delta \le \epsilon/5$. **35** ± 4 **37** 0, 2
39 $\mathbb{R}$ **41** $[-3,-2) \cup (-2,2) \cup (2,3]$
43 $\lim_{x\to 8} f(x) = 7 = f(8)$

45 (a)

x	$\frac{x^3+2x^2-9x-18}{x-3}$
2.99	29.8901
2.999	29.989001
2.9999	29.99890001
3.0001	30.00110001
3.001	30.011001
3.01	30.1101

47 (a)

x	$\frac{\cos(\pi x)}{x-(3/2)}$
1.45	3.1286893
1.495	3.1414635
1.4995	3.1415914
1.5005	3.1415914
1.505	3.1414635
1.55	3.1286893

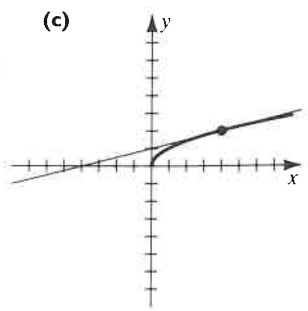
49 -0.874, 1.941 **51** $x \approx -1.618$, 0.618

CHAPTER ▪ 2

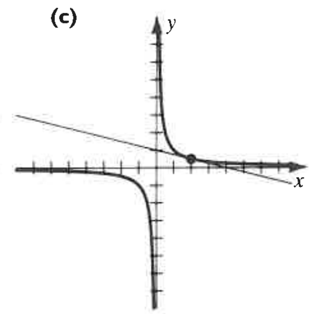
Exercises 2.1

1 (a) $10a-4$ **(b)** $y = 16x - 20$
3 (a) $3a^2$ **(b)** $y = 12x - 16$
5 (a) 3 **(b)** $y = 3x + 2$

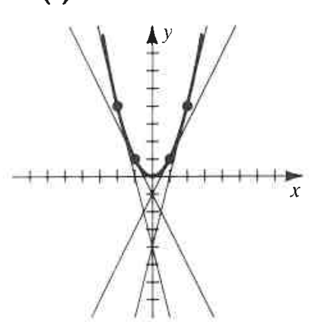
7 (a) $\frac{1}{2\sqrt{a}}$
(b) $y = \frac{1}{4}x + 1$
(c)



9 (a) $-\frac{1}{a^2}$
(b) $y = -\frac{1}{4}x + 1$
(c)



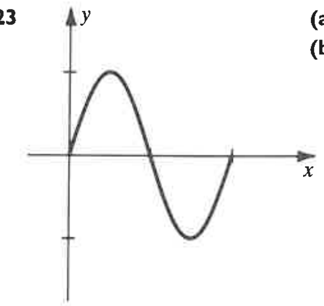
11 (a)



(b) (3, 9)

13 In cm/sec: **(a)** 11.8; 11.4; 11.04 **(b)** 11
15 In ft/sec: **(a)** -32 **(b)** $-32\sqrt{10}$
17 (a) Creature at $x = 3$ **(b)** No hit
19 (a) 6.5 **(b)** 6 **21 (a)** $p_v = -\frac{200}{v^2}$ **(b)** -2

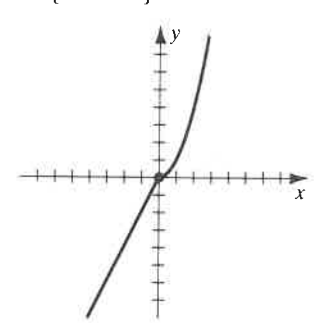
23 **(a)** -1
(b) -0.9703; -0.9713

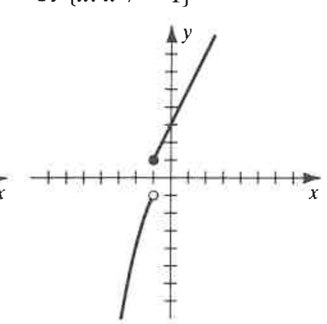


25 In ft/sec: -0.06864; -0.06426; -0.06382
27 (a) -1.851, -2.986, -0.966
29 (a) 1.129, -0.253, -0.500
31 (a) 0.322; 0.341; 0.360
(b) -0.222; -0.239; -0.255

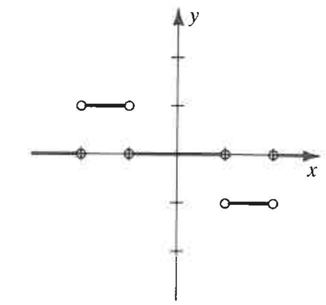
Exercises 2.2

1 (a) $-10x+8$ **(b)** $\mathbb{R}$ **(c)** $y = 18x + 7$
(d) $\left(\frac{4}{5}, \frac{26}{5}\right)$
3 (a) $3x^2+1$ **(b)** $\mathbb{R}$ **(c)** $y = 4x - 2$ **(d)** None
5 (a) 9 **(b)** $\mathbb{R}$ **(c)** $y = 9x - 2$ **(d)** None
7 (a) 0 **(b)** $\mathbb{R}$ **(c)** $y = 37$ **(d)** All
9 (a) $\frac{-3}{x^4}$ **(b)** $(-\infty, 0) \cup (0, \infty)$ **(c)** $y = -\frac{3}{16}x + \frac{1}{2}$
(d) None
11 (a) $\frac{1}{x^{3/4}}$ **(b)** $(0, \infty)$ **(c)** $y = \frac{1}{27}x + 9$ **(d)** None
13 $18x^5$; $90x^4$; $360x^3$ **15** $6x^{-1/3}$; $-2x^{-4/3}$; $\frac{8}{3}x^{-7/3}$
17 $36t^{-1/5}$ **19** 0
21 (a) No, because f is not differentiable at $x = 0$
(b) Yes, because f' exists for every number in $[1, 3)$
23 (a) No **(b)** Yes **25 (a)** Yes **(b)** No
27 (a) Yes **(b)** Yes **29 (a)** No **(b)** No
31 $f'(-1) = 1, f'(1) = 0, f'(2)$ is undefined, $f'(3) = -1$
33 The right-hand and left-hand derivatives are unequal at $a = 5$.
35 The right-hand and left-hand derivatives are unequal at $a = 2$.
37 $\{x: x \neq 0\}$ **39** $\{x: x \neq -1\}$





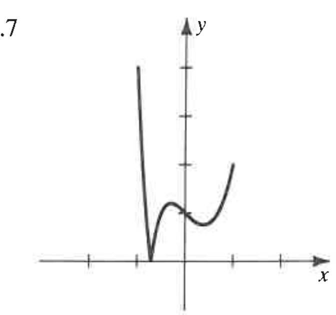
41 f is not differentiable at ± 1, ± 2.



43 (a) $f'(x) = \begin{cases} 6x - 6 & \text{if } 1 \le x < a \\ -14x + 54 & \text{if } a \le x < b \\ 16x - 96 & \text{if } b \le x \le 6 \end{cases}$

$f''(x) = \begin{cases} 6 & \text{if } 1 \le x < a \\ -14 & \text{if } a \le x < b \\ 16 & \text{if } b \le x \le 6 \end{cases}$

(b) f' exists at $x = a$ and at $x = b$.
45 $\frac{1}{8}$ **47** $F_C = \frac{9}{5}$ **49** $V_r = 4\pi r^2$
51 (a) $A_r = 2\pi r$ **(b)** 1000π ft²/ft
53 (a) The formula gives an approximation of the slope of the tangent line at $(a, f(a))$ by using the slope of the secant line through $P(a-h, f(a-h))$ and $Q(a+h, f(a+h))$.
(b) Subtract and add $f(a)$ in the numerator and consider two limits.
(c) -2.0406; -2.0004; -2.0000 **(d)** -2
55 (a) 53.2 ft/sec **(b)** 88.3 ft/sec
57 $x \approx -0.7$



59 (b) Horizontal: $x = 0$
61 (b) Horizontal: $x = 0$, $x \approx \pm 2.029$
63 (b) Not differentiable: $x = 0, \pm 1, 2$;
Horizontal: $x = 0.5$, $x \approx -0.618$, 1.618

Exercises 2.3

1 $10t^{2/3}$ **3** $-20s^3 + 8s - 1$ **5** $6x + \frac{4}{3}x^{1/3}$
7 $10x^4 + 9x^2 - 28x$ **9** $\frac{5}{2}x^{3/2} + \frac{3}{2}x^{1/2} - 2x^{-1/2}$
11 $18r^5 - 21r^2 + 4r$ **13** $416x^3 - 195x^2 + 64x - 20$

15 $\frac{23}{(3x+2)^2}$ **17** $\frac{-27z^2+12z+70}{(2-9z)^2}$ **19** $\frac{6v^2}{(v^3+1)^2}$

21 $-\frac{3t+10}{3\sqrt[3]{t}(3t-5)^2}$ **23** $-\frac{1+2x+3x^2}{(1+x+x^2+x^3)^2}$ **25** $\frac{-14x}{(x^2+5)^2}$

27 $2t-\frac{2}{t^3}$ **29** $-\frac{4}{81}s^{-5}$ **31** $10(5x-4)$ **33** $\frac{-10}{(5r-4)^3}$

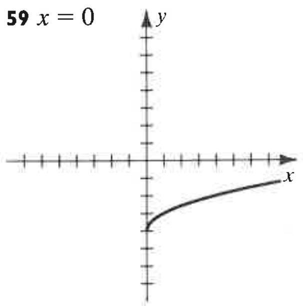
35 **(a)** $3x^2-10x+8$ **37** **(a)** $\frac{24x^4+8x+3}{x^4}$

39 **(a)** $\frac{12x^2+16x-13}{(3x+2)^2}$ **41** $-2, 3$ **43** $0, 4$ **45** $-5, 3$

47 $\frac{-3x+2}{x^3}$ **49** $\frac{4x-3}{3\sqrt[3]{x^2}}$ **51** $\frac{2}{(x+1)^3}$ **53** $y=\frac{4}{5}x+\frac{13}{5}$

55 **(a)** $-2, \frac{2}{3}$ **(b)** $-\frac{4}{3}, 0$ **57** $(1, 0)$

59 $x=0$



61 In ft/sec: **(a)** 4, 10, 18 **(b)** $6\sqrt{5}$

63 $-\frac{1}{2}$ **65** $y=2x-1$, $y=18x-81$

67 **(a)** 1 **(b)** -3 **(c)** -4 **(d)** 11 **(e)** $-\frac{1}{25}$ **(f)** $\frac{1}{9}$

69 **(a)** -4 **(b)** 1 **(c)** -20 **(d)** $-\frac{1}{4}$

73 $(8x-1)(x^2+4x+7)(3x^2)+$
$(8x-1)(2x+4)(x^3-5)+8(x^2+4x+7)(x^3-5)$

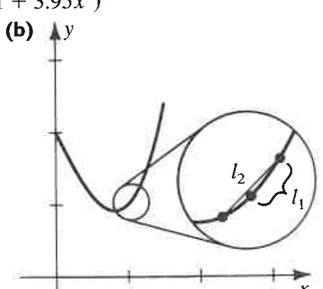
75 $x(2x^3-5x-1)(12x)+x(6x^2-5)(6x^2+7)+$
$(2x^3-5x-1)(6x^2+7)$

77 **(a)** $\frac{1}{4}$ cm/min **(b)** 36π cm³/min **(c)** 12π cm²/min

79 In cm²/sec: **(a)** 3200π **(b)** 6400π **(c)** 9600π

81 **(a)** $R=40$ when $x=0$ and R decreases to 6 as $x\to\infty$, since $dR/dx<0$ for $x>0$.
(b) $\frac{dR}{dx}=\frac{-537.2x^3}{(1+3.95x^4)^2}$ **(c)** $x\approx 0.624$

83 **(a)** 1.01 **(b)**



(c) 1; l_2 is nearly parallel to the tangent line, but l_1 is not.

Exercises 2.4

1 1 **3** $\frac{1}{8}$ **5** $\frac{2}{3}$ **7** 0 **9** $-\frac{3}{4}$ **11** 0 **13** 7

15 1 **17** 0 **19** 2 **21** 1 **23** 1 **25** -1

31 $-4\sin x$ **33** $5\csc v(1-v\cot v)$

35 $t^2\sin t-2t\cos t+1$ **37** $\frac{\theta\cos\theta-\sin\theta}{\theta^2}$

39 $t^2(t\cos t+3\sin t)$

41 $-2x\csc^2 x+2\cot x+x^2\sec^2 x+2x\tan x$

43 $\frac{2\sin z}{(1+\cos z)^2}$ **45** $-\csc x(1+2\cot^2 x)$

47 $-x\csc^2 x-\csc^3 x+\cot x-\csc x\cot^2 x$

49 $-\sin x$ **51** $\frac{\sec^2 x+x^2\sec^2 x-2x\tan x}{(1+x^2)^2}$ **53** $-\csc^2 v$

55 $-\cos x-\sin x$ **57** $\sin\phi+\sec\phi\tan\phi$

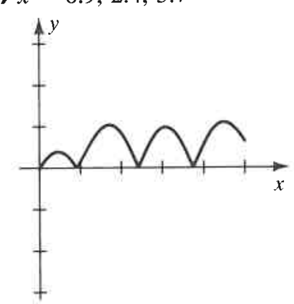
59 $y-\sqrt{2}=\sqrt{2}\left(x-\frac{\pi}{4}\right)$; $y-\sqrt{2}=-\frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right)$

61 $\left(\frac{\pi}{4}, \sqrt{2}\right), \left(\frac{5\pi}{4}, -\sqrt{2}\right)$ **63** $\left(\frac{\pi}{4}, 2\sqrt{2}\right)$

65 **(a)** $\frac{\pi}{6}+2\pi n, \frac{5\pi}{6}+2\pi n$ **(b)** $y=x+2$

67 **(a)** $\frac{\pi}{4}+2\pi n, \frac{7\pi}{4}+2\pi n$ **(b)** $y-4=\sqrt{3}\left(x-\frac{\pi}{6}\right)$

69 $x\approx 0.9, 2.4, 3.7$



71 $\frac{\pi}{6}+2\pi n, \frac{5\pi}{6}+2\pi n$

73 $(16, 96)$

75 **(a)** $-\sin x$; $-\cos x$; $\sin x$; $\cos x$ **(b)** $\sin x$

77 $2\sec^2 x(3\tan^2 x+1)$

79 $D_x\cot x=D_x\left(\frac{\cos x}{\sin x}\right)=\frac{(\sin x)(-\sin x)-(\cos x)(\cos x)}{\sin^2 x}$
$=\frac{-1(\sin^2 x+\cos^2 x)}{\sin^2 x}=-\frac{1}{\sin^2 x}=-\csc^2 x$

81 $D_x\sin 2x=D_x(2\sin x\cos x)$
$=2[\sin x(-\sin x)+\cos x\cos x]$
$=2(\cos^2 x-\sin^2 x)=2\cos 2x$

Exercises 2.5

1 $6x^2(x^3-4)$ **3** $\frac{-3}{2(3x-2)^{3/2}}$ **5** $6x\sec^2(3x^2)$

7 $3(x^2-3x+8)^2(2x-3)$ **9** $-40(8x-7)^{-6}$

11 $-\frac{7x^2+1}{(x^2-1)^5}$

13 $5(8x^3-2x^2+x-7)^4(24x^2-4x+1)$

15 $17{,}000(17v-5)^{999}$

17 $2(6x-7)^2(8x^2+9)(168x^2-112x+81)$

19 $12\left(z^2-\frac{1}{z^2}\right)^5\left(z+\frac{1}{z^3}\right)$ **21** $8r^2(8r^3+27)^{-2/3}$

23 $-5v^4(v^5-32)^{-6/5}$ **25** $\frac{w^2+4w-9}{2w^{5/2}}$ **27** $\frac{6(3-2x)}{(4x^2+9)^{3/2}}$

29 $2x\cos(x^2+2)$ **31** $-15\cos^4 3\theta\sin 3\theta$

33 $4(2z+1)\sec(2z+1)^2\tan(2z+1)^2$

35 $(2-3s^2)\csc^2(s^3-2s)$

37 $-6x\sin(3x^2)-6\cos 3x\sin 3x$

39 $-4\csc^2 2\phi\cot 2\phi$ **41** $2z\cot 5z-5z^2\csc^2 5z$

43 $2\tan\theta\sec^5\theta+3\tan^3\theta\sec^3\theta$

45 $25(\sin 5x-\cos 5x)^4(\cos 5x+\sin 5x)$

47 $-9\cot^2(3w+1)\csc^2(3w+1)$ **49** $\frac{4}{1-\sin 4w}$

51 $6\tan 2x\sec^2 2x(\tan 2x-\sec 2x)$

53 $\frac{\cos\sqrt{x}}{2\sqrt{x}}+\frac{\cos x}{2\sqrt{\sin x}}$

55 $\frac{8\cos\sqrt{3-8\theta}\sin\sqrt{3-8\theta}}{\sqrt{3-8\theta}}$

57 $x\sec^2\sqrt{x^2+1}+\frac{x\tan\sqrt{x^2+1}}{\sqrt{x^2+1}}$

59 $\frac{2\sec\sqrt{4x+1}\tan\sqrt{4x+1}}{\sqrt{4x+1}}$ **61** $-\frac{3\csc^2 3x\cot 3x}{\sqrt{4+\csc^2 3x}}$

63 **(a)** $y-81=864(x-2)$; $y-81=-\frac{1}{864}(x-2)$
(b) $\frac{1}{2}, 1, \frac{3}{2}$

65 **(a)** $y=32$; $x=1$ **(b)** ± 1

67 **(a)** $y=6x$; $y=-\frac{1}{6}x$ **(b)** $\frac{\pi}{3}+\frac{2\pi}{3}n$

69 $\frac{3}{2(3z+1)^{1/2}}; -\frac{9}{4(3z+1)^{3/2}}$

71 $20(4r+7)^4$; $320(4r+7)^3$

73 $3\sin^2 x\cos x$; $6\sin x\cos^2 x-3\sin^3 x$

75 $\frac{dK}{dt}=mv\frac{dv}{dt}$ **77** -0.1819 lb/sec **79** -4; 15

81 $-\frac{2}{5}$ **83** 4.91

85 **(a)** *Hint:* Differentiate both sides of $f(-x)=f(x)$ using the chain rule.
(b) *Hint:* Differentiate both sides of $f(-x)=-f(x)$ using the chain rule.

87 **(a)** $\frac{dW}{dt}=(1.644\times 10^{-4})L^{1.74}\frac{dL}{dt}$
(b) 7.876 cm/month

89 $\frac{dd}{ds}=60cs-3cs^2$

91 **(b)** $\frac{dL}{d\theta}=\frac{b}{8}\sec^2\left(\frac{\theta}{4}\right)$

Exercises 2.6

1 $-\frac{8x}{y}$ **3** $-\frac{6x^2+2xy}{x^2+3y^2}$ **5** $\frac{10x-y}{x+8y}$ **7** $-\sqrt{\frac{y}{x}}$

9 $\frac{-4x\sqrt{xy}-y}{x}$ **11** $\frac{1}{6\sin 3y\cos 3y-1}=\frac{1}{3\sin 6y-1}$

13 $\frac{-y\cot(xy)\csc(xy)}{1+x\cot(xy)\csc(xy)}$ **15** $\frac{\cos y}{x\sin y+2y}$

17 $\frac{4x\sqrt{\sin y}}{4y\sqrt{\sin y}-\cos y}$ **19** $-\frac{\sqrt{2}}{5}$ **21** -1 **23** 4

25 $-\frac{36}{23}$ **27** -2π **29** $-\frac{3}{4y^3}$ **31** $-\frac{2x}{y^5}$

33 $\frac{\sin y}{(1+\cos y)^3}$ **35** An infinite number **37** None

39 Let $f_c(x)=\begin{cases}\sqrt{x} & \text{if } 0\le x\le c\\ -\sqrt{x} & \text{if } x>c\end{cases}$ for any $c>0$

41 $\left(\pm\sqrt{3}, \frac{3}{2}\right)$ **43** $y-3=\frac{5}{6}(x+2)$

45 $5x-6y=-28$; $6x+5y=3$

47 $4x+5y=-3$; $5x-4y=-14$

49 **(a)** $y'=-\frac{x_1b^2}{y_1a^2}$
(b) Horizontal: $(0, \pm b)$; vertical: $(\pm a, 0)$

Exercises 2.7

1 60 **3** $\frac{4}{15}$ **5** 3 **7** $-\frac{24}{17}$

9 $0.15\pi\approx 0.471$ cm²/min **11** $\frac{20}{9\pi}\approx 0.707$ ft/min

13 $-\frac{3}{8}\sqrt{336}\approx -6.9$ ft/sec **15** $\frac{64}{11}$ ft/sec; $\frac{20}{11}$ ft/sec

17 $-7442\pi\approx -23{,}380$ in³/hr **19** $\frac{10}{3}$ ft/sec

21 5 in³/min (increasing) **23** $\frac{15}{32}\sqrt{3}\approx 0.81$ ft/min

25 $-\frac{\sqrt{2}}{5\sqrt[4]{3}}\approx -0.2149$ cm/min **27** π m/sec

29 $\frac{11}{1600}=0.006875$ ohm/sec **31** $\frac{13.37}{112\pi}\approx 0.038$ ft/min

33 64 ft/sec **35** $\frac{180(6+\sqrt{2})}{\sqrt{10+3\sqrt{2}}}\approx 353.6$ mi/hr

37 $-\frac{27}{25\pi}\approx -0.3438$ in./hr **39** $\frac{10{,}000\pi}{135}\approx 232.7$ ft/sec

41 $\frac{\pi}{10}\sqrt{3}\approx 0.54$ in²/min

43 $\frac{2{,}640{,}000}{\sqrt{180{,}400}}\approx 6215.6$ ft/min ≈ 70.63 mi/hr

45 $\frac{1000\pi}{3}$ ft/sec ≈ 714.0 mi/hr

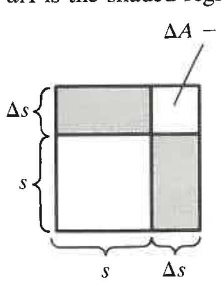
47 Ground speed is $\frac{175}{88}\frac{d\theta}{dt}$ mi/hr.

49 (a) $2v\frac{dv}{dt} = gr(1+\sec^2\theta)\frac{d\theta}{dt}$

(b) $2v\frac{dv}{dt} = g\tan\theta\,(1+\sec^2\theta)\frac{dr}{dt}$ **51** 19.25 mi/hr

Exercises 2.8

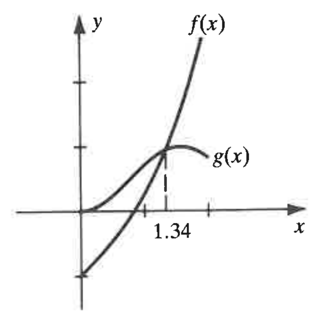
1 −3.94 **3** 0.92 **5** 1.80 **7** 2.12
9 (a) $(4x-4)\Delta x + 2(\Delta x)^2$; $(4x-4)dx$
(b) −0.72; −0.8
11 (a) $\frac{-(2x+\Delta x)\Delta x}{x^2(x+\Delta x)^2}$; $-\frac{2}{x^3}dx$
(b) $-\frac{7}{363}\approx -0.01928$; $-\frac{1}{45} = -0.0\overline{2}$
13 (a) $-9\,\Delta x$ **(b)** $-9\,dx$ **(c)** 0
15 (a) $(6x+5)\,\Delta x + 3(\Delta x)^2$ **(b)** $(6x+5)\,dx$
(c) $-3(\Delta x)^2$
17 (a) $\frac{-\Delta x}{x(x+\Delta x)}$ **(b)** $-\frac{1}{x^2}dx$ **(c)** $\frac{-(\Delta x)^2}{x^2(x+\Delta x)}$
19 (a) With $h = 0.001$, $y \approx -0.98451 - 0.27315(x-2.5)$.
(b) −1.011825 **(c)** −1.011825
(d) They are equal because the tangent line approximation is equivalent to using (2.35).
21 (a) 4.0208 **(b)** 4.0207 **23 (a)** 3.666 **(b)** 3.659
25 (a) 0.51511 **(b)** 0.51504
27 ±0.02; ±2% **29** ±0.04; ±4% **31** 1.1
33 ±45% **35** ±0.06
37 $\pm1.92\pi$ in² ≈ ±6.03 in²; ±0.0075; ±0.75%
39 30 in³; 30.301 in³
41 3301.661 ft²; ±11.464 ft²; ±0.00347; ±0.347%
43 $\frac{1}{50\pi}$ cm ≈ 0.00637 cm **45** −1 cm **47** 40% increase
49 $\frac{5\sqrt{2\pi}}{81}$ lb ≈ 0.274 lb **51** $\pm\frac{\pi}{9}$ ft ≈ ±0.35 ft **53** ±0.19°
55 *Hint:* Show that $v\,dp = -\frac{c}{v}\,dv$.
57 dA is the shaded region.



59 (a) 60π cm²; ±1.508 cm² **(b)** ±0.8%
61 0.09 **63 (a)** 1.28 **(b)** $c \approx 1.25$

Exercises 2.9

1 (a) 3.3166 **(b)** 3.3166 **3** 1.2599 **5** 1.3315
7 −1.7321 **9** 4.6458 **11** 0.56 **13** 1.50
15 ±3.34 **17** −1, 1.35 **19** −1.88, 0.35, 1.53
21 2.71 **23** −1.16, 1.45 **25** ±2.99
27 (a) 3, 3.1425465, 3.1415927, 3.1415926, 3.1415926
(b) They approach 2π.
29 $f'\left(\frac{1}{2}\right) = 0$ and hence the expression for x_2 would be undefined.
31 (a) f: $x_1 = 1.1$, $x_2 = 1.066485$, $x_3 = 1.044237$, $x_4 = 1.029451$
g: $x_1 = 1.1$, $x_2 = 0.9983437$, $x_3 = 0.9999995$, $x_4 = 1.000000$
(b) Because $f'(1) = 0$
33 $x_5 = 0.525$
35 (a) 1.5 **(b)** 1.34



37 (a) $x_1 = 2$, $x_{12} \approx x_{13} \approx 1.93456$
(b) $x_1 = 0.5$, $x_8 \approx x_9 \approx 0.45018$

Chapter 2 Review Exercises

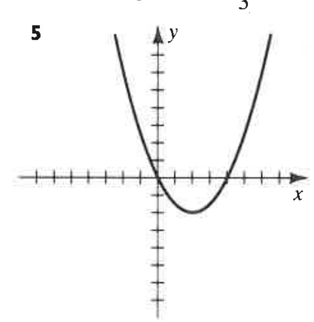
1 $\frac{-24x}{(3x^2+2)^2}$ **3** $6x^2-7$ **5** $\frac{3}{\sqrt{6t+5}}$
7 $\frac{2(7z-2)}{3(7z^2-4z+3)^{2/3}}$ **9** $-\frac{144x}{(3x^2-1)^5}$ **11** $-\frac{4(r+r^{-3})}{(r^2-r^{-2})^3}$
13 $\frac{12}{5(3x+2)^{1/5}}$ **15** $\frac{1024s(2s^2-1)^3(18s^3-27s+4)}{(1-9s^3)^5}$
17 $3(x^6+1)^4(3x+2)^2(33x^6+20x^5+3)$
19 $(9s-1)^3(108s^2-139s+39)$ **21** $12x+\frac{5}{x^2}-\frac{4}{3x^{5/3}}$
23 $\frac{-53}{2\sqrt{(2w+5)(7w-9)^3}}$ **25** 0 **27** $\frac{2}{3}$ **29** $\frac{3}{5}$ **31** 2
33 $-\frac{\sin 2r}{\sqrt{1+\cos 2r}}$
35 $12x^2\sin 8x^3$ **37** $5\sec x(\sec x+\tan x)^5$
39 $2x(\cot 2x - x\csc^2 2x)$ **41** $\frac{2}{1+\cos 2\theta}$
43 $-\frac{(\cos\sqrt[3]{x}-\sin\sqrt[3]{x})^2(\cos\sqrt[3]{x}+\sin\sqrt[3]{x})}{\sqrt[3]{x^2}}$
45 $\frac{\csc u(1-\cot u+\csc u)}{(\cot u+1)^2}$ **47** $10\tan 5x\sec^2 5x$
49 $\frac{\tan^3(\sqrt[4]{\theta})\sec^2(\sqrt[4]{\theta})}{\sqrt[4]{\theta^3}}$ **51** $\frac{4xy^2-15x^2}{12y^2-4x^2y}$

53 $\frac{1}{\sqrt{x}(3\sqrt{y}+2)}$
55 $\frac{\cos(x+2y)-y^2}{2xy-2\cos(x+2y)}$ **57** $y=\frac{9}{4}x-3$; $y=-\frac{4}{9}x+\frac{70}{9}$
59 $\frac{7\pi}{12}+\pi n, \frac{11\pi}{12}+\pi n$
61 $15x^2+\frac{2}{\sqrt{x}}$; $30x-\frac{1}{\sqrt{x^3}}$; $30+\frac{3}{2\sqrt{x^5}}$
63 $\frac{5(y^2-4xy-x^2)}{(y-2x)^3} = -\frac{40}{(y-2x)^3}$
65 (a) $6x\,\Delta x + 3(\Delta x)^2$ **(b)** $6x\,dx$ **(c)** $-3(\Delta x)^2$
67 $\pm0.06\sqrt{3}\approx\pm0.104$ in²; ±1.5% **69** −0.57
71 (a) 2 **(b)** −7 **(c)** −14 **(d)** 21 **(e)** $-\frac{10}{9}$
(f) $-\frac{19}{27}$
73 (a) Vertical tangent line at (−1, −4)
(b) Cusp at (8, −1)
75 2% **77** $\frac{68\pi}{5}$ ft²/ft **79** $\frac{5}{6}$ ft³/min **81** $\frac{dp}{dv} = -\frac{p}{v}$
83 (a) $h(t) = 60 - 50\cos\frac{\pi}{15}t$ **(b)** 10.4 ft/sec
85 4.493

CHAPTER 3

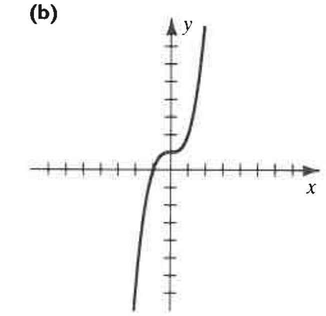
Exercises 3.1

1 Maximum of 4 at 2; minimum of 0 at 4; local maximum at $x = 2$, $6 \le x \le 8$; local minimum at $x = 4$, $6 < x < 8$, $x = 10$
3 (a) Min: $f(-3) = -6$; max: none
(b) Min: none; max: $f(-1) = \frac{2}{3}$
(c) Min: none; max: $f(-1) = \frac{2}{3}$
(d) Min: $f(1) = -\frac{2}{3}$; max: $f(3) = 6$
5



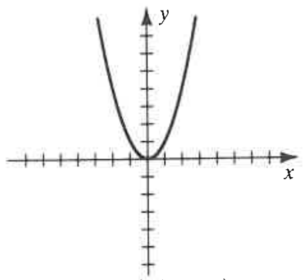
(a) Min: $f(2) = -2$; max: none
(b) Min: none; max: none
(c) Min: $f(2) = -2$; max: none
(d) Min: $f(2) = -2$; max: $f(5) = \frac{5}{2}$

7 Min: $f(-2) = f(1) = -3$; max: $f(-3) = f(0) = 5$
9 Min: $f(8) = -3$; max: $f(0) = 1$ **11** $\frac{3}{8}$ **13** $-2, \frac{5}{3}$
15 2 **17** ±4 **19** $\frac{5+\sqrt{153}}{8}, \pm 2$ **21** $0, \frac{15}{7}, \frac{5}{2}$
23 None **25** $\pi n, \frac{2\pi}{3}+2\pi n, \frac{4\pi}{3}+2\pi n$
27 $\frac{\pi}{6}+2\pi n, \frac{5\pi}{6}+2\pi n, \frac{3\pi}{2}+2\pi n$ **29** $\frac{3\pi}{2}+2\pi n$
31 πn **33** None
35 $0, \pm\sqrt{k\pi-1}$ for $k = 1, 2, 3, \ldots$
37 (a) Since $f'(x) = \frac{1}{3}x^{-2/3}$, $f'(0)$ does not exist. If $a \ne 0$, then $f'(a) \ne 0$. Hence, 0 is the only critical number of f. The number $f(0) = 0$ is not a local extremum, since $f(x) < 0$ if $x < 0$ and $f(x) > 0$ if $x > 0$.
(b) The only critical number is 0, for the same reasons given in part (a). The number $f(0) = 0$ is a local minimum, since $f(x) > 0$ if $x \ne 0$.
39 (a) There is a critical number, 0, but $f(0)$ is not a local extremum, since $f(x) < f(0)$ if $x < 0$ and $f(x) > f(0)$ if $x > 0$.
(b)

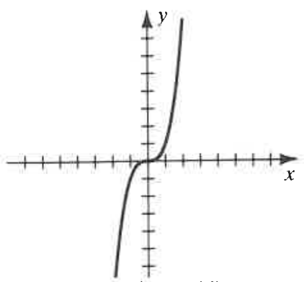


(c) The function is continuous at every number a, since $\lim_{x\to a} f(x) = f(a)$. If $0 < x_1 < x_2 < 1$, then $f(x_1) < f(x_2)$ and hence there is neither a maximum nor a minimum on (0, 1).
(d) This does not contradict Theorem (3.3) because the interval (0, 1) is open.
41 (a) If $f(x) = cx + d$ and $c \ne 0$, then $f'(x) = c \ne 0$. Hence, there are no critical numbers.
(b) On $[a, b]$, the function has absolute extrema at a and b.
43 If $x = n$ is an integer, then $f'(n)$ does not exist. Otherwise, $f'(x) = 0$ for every $x \ne n$.
45 If $f(x) = ax^2 + bx + c$ and $a \ne 0$, then $f'(x) = 2ax + b$. Hence, $-b/(2a)$ is the only critical number of f.
47 Since $f'(x) = nx^{n-1}$, the only possible critical number is $x = 0$, and $f(0) = 0$. If n is even, then $f(x) > 0$ if $x \ne 0$ and hence 0 is a local minimum. If n is odd, then 0 is

not an extremum, since $f(x) < 0$ if $x < 0$ and $f(x) > 0$ if $x > 0$.

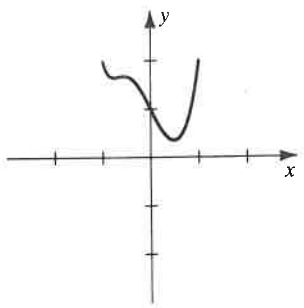


$y = x^n$ (n is even)

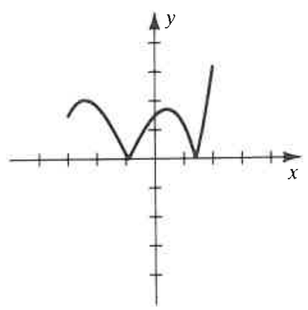


$y = x^n$ (n is odd)

49 Min: $f(0.48) \approx 0.36$; max: $f(-1) = f(1) = 2$



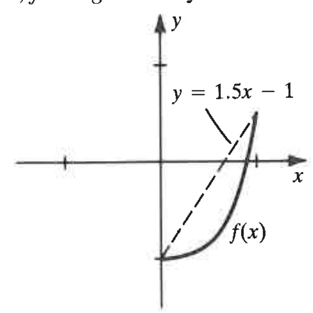
51 $-2.41, -0.92, 0.41, 1.41$



53 $-1.662, 0, 2.175$
55 $0.131, 2.535, 3, 4$
57 $-0.222, 0, 0.818, 15.404$

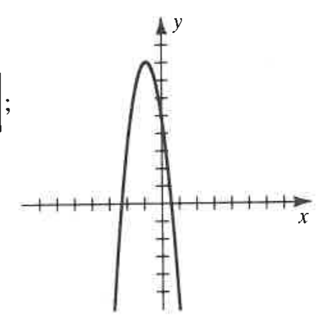
Exercises 3.2

1 3, 7 **3** 2 **5** 0 **7** $\frac{\pi}{4}, \frac{3\pi}{4}$ **9** 2
11 f is not continuous on $[0, 2]$.
13 f is not differentiable on $(-8, 8)$. **15** 2
17 $\frac{1}{3}(2 - \sqrt{7}) \approx -0.22$ **19** 2 **21** 2
23 The number c such that $\cos c = 2/\pi$ ($c \approx 0.88$)
25 $-0.371, 1.307$
27 -0.5
29 4.6926
31 f is not differentiable on $(1, 4)$.
33 $f(-1) = f(1) = 1$. $f'(x) = 1$ if $x > 0$, $f'(x) = -1$ if $x < 0$, and $f'(0)$ does not exist. This does not contradict Rolle's theorem, because f is not differentiable throughout the open interval $(-1, 1)$.
35 *Hint:* Show that $c^2 = -4$.
37 *Hint:* Let $f(x) = px + q$.
39 *Hint:* If f has degree 3, then $f'(x)$ is a polynomial of degree 2.
41 Let x be any number in $(a, b]$. Applying the mean value theorem to the interval $[a, x]$ yields $f(x) - f(a) = f'(c)(x - a) = 0(x - a) = 0$. Thus, $f(x) = f(a)$, and hence f is a constant function.
43 *Hint:* Use the method of Example 4.
45 *Hint:* Show that $dW/dt < -44$ lb/mo.
49 *Hint:* Show that $dI/dt > 3500$ cases/mo.
51 $-1/x + C$
53 $\sin x + C$
55 **(a)** $f' = g' = 2 \sin x \cos x$
(b) No, f and g differ by a constant.
57 0.64

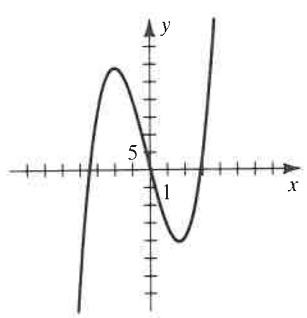


Exercises 3.3

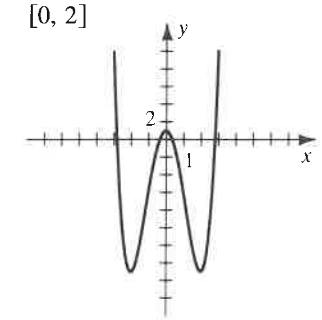
1 Max: $f\left(-\frac{7}{8}\right) = \frac{129}{16}$; increasing on $\left(-\infty, -\frac{7}{8}\right]$; decreasing on $\left[-\frac{7}{8}, \infty\right)$



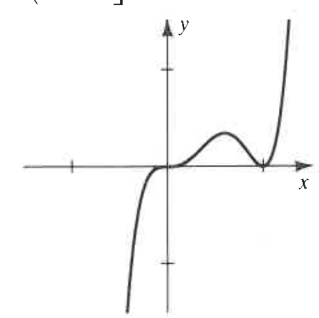
3 Max: $f(-2) = 29$; min: $f\left(\frac{5}{3}\right) = -\frac{548}{27}$; increasing on $(-\infty, -2]$ and $\left[\frac{5}{3}, \infty\right)$; decreasing on $\left[-2, \frac{5}{3}\right]$



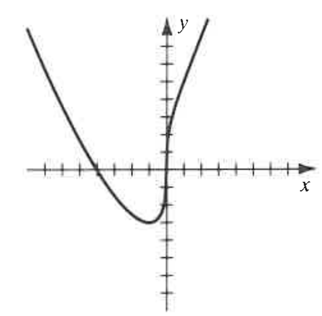
5 Max: $f(0) = 1$; min: $f(-2) = f(2) = -15$; increasing on $[-2, 0]$ and $[2, \infty)$; decreasing on $(-\infty, -2]$ and $[0, 2]$



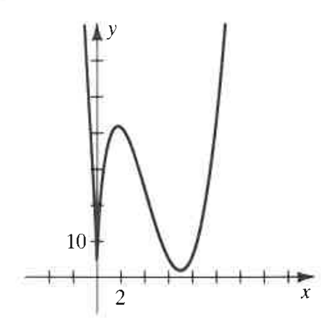
7 Max: $f\left(\frac{3}{5}\right) = \frac{216}{625} \approx 0.35$; min: $f(1) = 0$; increasing on $\left(-\infty, \frac{3}{5}\right]$ and $[1, \infty)$; decreasing on $\left[\frac{3}{5}, 1\right]$



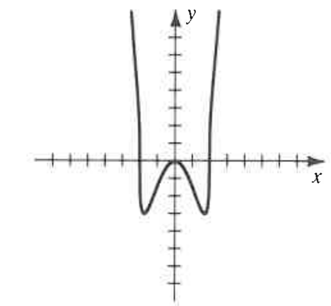
9 Min: $f(-1) = -3$; increasing on $[-1, \infty)$; decreasing on $(-\infty, -1]$



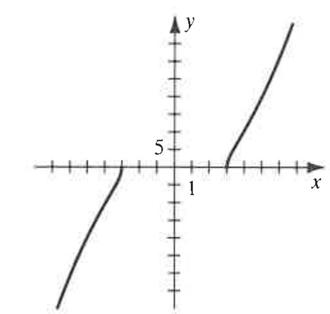
11 Max: $f\left(\frac{7}{4}\right) = \frac{441}{16}\sqrt[3]{\frac{49}{16}} + 2 \approx 42.03$; min: $f(0) = f(7) = 2$; increasing on $\left[0, \frac{7}{4}\right]$ and $[7, \infty)$; decreasing on $(-\infty, 0]$ and $\left[\frac{7}{4}, 7\right]$



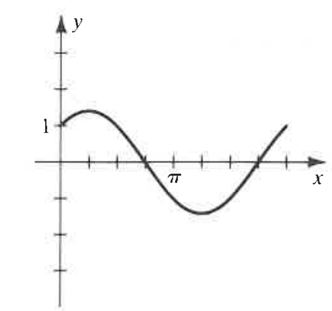
13 Max: $f(0) = 0$; min: $f(-\sqrt{3}) = f(\sqrt{3}) = -3$; increasing on $[-\sqrt{3}, 0]$ and $[\sqrt{3}, \infty)$; decreasing on $(-\infty, -\sqrt{3}]$ and $[0, \sqrt{3}]$



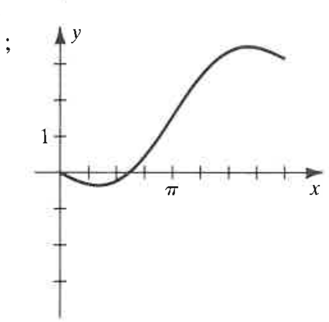
15 No extrema; increasing on $(-\infty, -3]$ and $[3, \infty)$



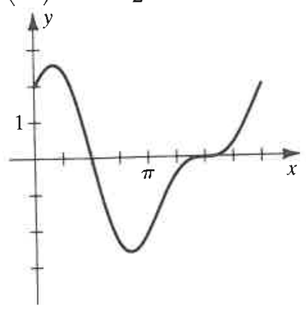
17 Max: $f\left(\frac{\pi}{4}\right) = \sqrt{2}$; min: $f\left(\frac{5\pi}{4}\right) = -\sqrt{2}$; increasing on $\left[0, \frac{\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, 2\pi\right]$; decreasing on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$



19 Max: $f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2}$; min: $f\left(\frac{\pi}{3}\right) = \frac{\pi}{6} - \frac{\sqrt{3}}{2}$; increasing on $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$; decreasing on $\left[0, \frac{\pi}{3}\right]$ and $\left[\frac{5\pi}{3}, 2\pi\right]$



21 Max: $f\left(\frac{\pi}{6}\right)=\frac{3\sqrt{3}}{2}$; min: $f\left(\frac{5\pi}{6}\right)=-\frac{3\sqrt{3}}{2}$;
increasing on $\left[0, \frac{\pi}{6}\right]$
and $\left[\frac{5\pi}{6}, 2\pi\right]$;
decreasing on $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$



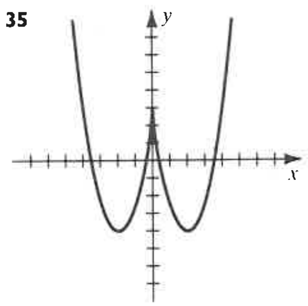
23 Max: $f(-\sqrt{3})=\sqrt[3]{6\sqrt{3}}\approx 2.18$;
min: $f(\sqrt{3})=-\sqrt[3]{6\sqrt{3}}$

25 Max: $f(-1)=0$; min: $f\left(\frac{5}{7}\right)=-\frac{9^3 12^4}{7^7}\approx -18.36$

27 Max: $f(4)=\frac{1}{16}$ **29** Min: $f(0)=1$

31 Max: $f\left(\frac{\pi}{4}\right)=1$ **33** $-\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

35



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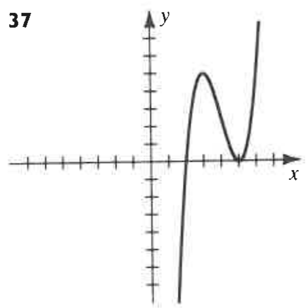

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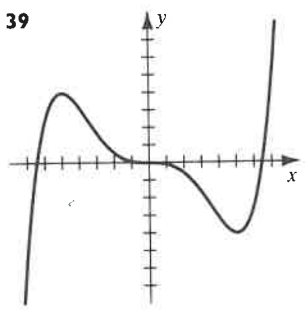

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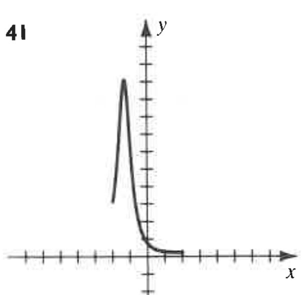
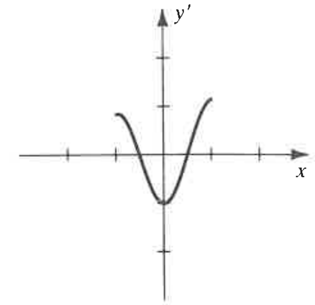


(a) Max: $f(-1.31)\approx 10.13$
(b) increasing on $[-2, -1.31]$; decreasing on $[-1.31, 2]$

43 (a) Max: $f(2.55)\approx 105.63$; min: $f(0.78)\approx 102.89$, $f(6.35)\approx 12.69$
(b) Increasing on $[0.78, 2.55]$ and $[6.35, 10]$; decreasing on $[-4, 0.78]$ and $[2.55, 6.35]$

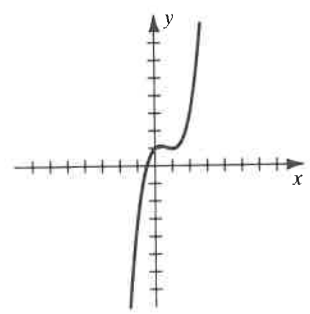
45 Max at $x\approx -0.51$;
min at $x\approx 0.49$



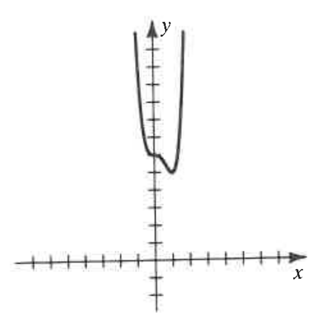
47 Max at $x\approx 0.46, 1.78, 5.97$; min at $x\approx 1.03, 5.22$

Exercises 3.4

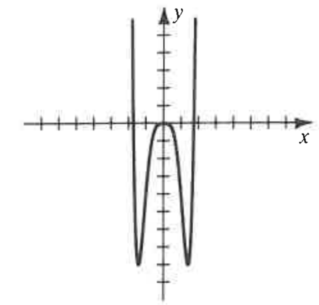
1 Since $f''\left(\frac{1}{3}\right)=-2<0$, $f\left(\frac{1}{3}\right)=\frac{31}{27}$ is a maximum; since $f''(1)=2>0$, $f(1)=1$ is a minimum; CU on $\left(\frac{2}{3}, \infty\right)$; CD on $\left(-\infty; \frac{2}{3}\right)$; x-coordinate of PI is $\frac{2}{3}$.



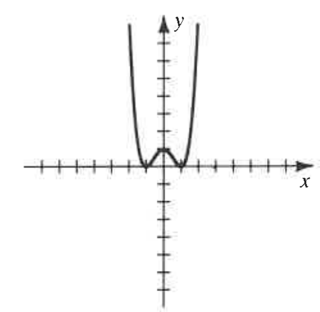
3 Since $f''(1)=12>0$, $f(1)=5$ is a minimum; CU on $(-\infty, 0)$ and $\left(\frac{2}{3}, \infty\right)$; CD on $\left(0, \frac{2}{3}\right)$; x-coordinates of PI are 0 and $\frac{2}{3}$.



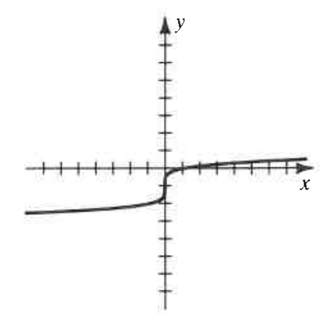
5 Since $f''(0)=0$, use the first derivative test to show that $f(0)=0$ is a maximum; since $f''(\pm\sqrt{2})=96>0$, $f(\pm\sqrt{2})=-8$ are minima; CU on $\left(-\infty, -\sqrt{\frac{6}{5}}\right)$ and $\left(\sqrt{\frac{6}{5}}, \infty\right)$; CD on $\left(-\sqrt{\frac{6}{5}}, \sqrt{\frac{6}{5}}\right)$; x-coordinates of PI are $\pm\sqrt{\frac{6}{5}}$.



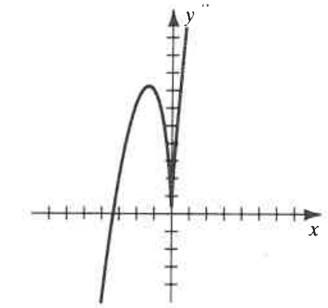
7 Since $f''(0)=-4<0$, $f(0)=1$ is a maximum; since $f''(\pm 1)=8>0$, $f(\pm 1)=0$ are minima; CU on $\left(-\infty, -\sqrt{\frac{1}{3}}\right)$ and $\left(\sqrt{\frac{1}{3}}, \infty\right)$; CD on $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$; x-coordinates of PI are $\pm\sqrt{\frac{1}{3}}$.



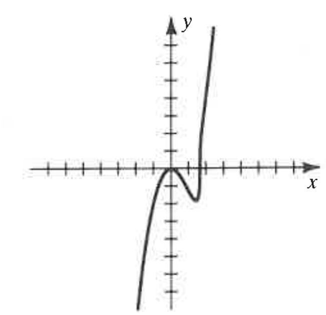
9 No local extrema; CU on $(-\infty, 0)$; CD on $(0, \infty)$; x-coordinate of PI is 0.



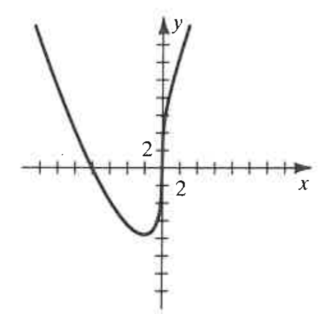
11 Since $f''\left(-\frac{4}{3}\right)<0$, $f\left(-\frac{4}{3}\right)\approx 7.27$ is a maximum; since $f''(0)$ is undefined, use the first derivative test to show that $f(0)=0$ is a minimum; CU on $\left(\frac{2}{3}, \infty\right)$; CD on $(-\infty, 0)$ and $\left(0, \frac{2}{3}\right)$; x-coordinate of PI is $\frac{2}{3}$.



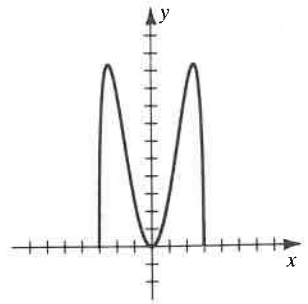
13 Since $f''(0)<0$, $f(0)=0$ is a maximum; since $f''\left(\frac{10}{7}\right)>0$, $f\left(\frac{10}{7}\right)\approx -1.82$ is a minimum. Let $a=\frac{20-5\sqrt{2}}{14}\approx 0.92$ and $b=\frac{20+5\sqrt{2}}{14}\approx 1.93$. CU on $\left(a, \frac{5}{3}\right)$ and (b, ∞); CD on $(-\infty, a)$ and $\left(\frac{5}{3}, b\right)$; x-coordinates of PI are a, $\frac{5}{3}$, and b.



15 Since $f''(-2)>0$, $f(-2)\approx -7.55$ is a minimum; CU on $(-\infty, 0)$ and $(4, \infty)$; CD on $(0, 4)$; x-coordinates of PI are 0 and 4.



17 Since $f''(\pm\sqrt{6}) < 0$, $f(\pm\sqrt{6}) \approx 10.4$ are maxima; since $f''(0) > 0$, $f(0) = 0$ is a minimum. Let $a = -\frac{1}{2}\sqrt{27 - 3\sqrt{33}} \approx -1.56$ and $b = -a$. CU on (a, b); CD on $(-3, a)$ and $(b, 3)$; x-coordinates of PI are a and b.



19 Since $f''\left(\frac{\pi}{4}\right) = -\sqrt{2} < 0$, $f\left(\frac{\pi}{4}\right) = \sqrt{2}$ is a maximum; since $f''\left(\frac{5\pi}{4}\right) = \sqrt{2} > 0$, $f\left(\frac{5\pi}{4}\right) = -\sqrt{2}$ is a minimum.

21 Since $f''\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2} < 0$, $f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2}$ is a maximum; since $f''\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} > 0$, $f\left(\frac{\pi}{3}\right) = \frac{\pi}{6} - \frac{\sqrt{3}}{2}$ is a minimum.

23 Since $f''\left(\frac{\pi}{6}\right) = -3\sqrt{3} < 0$, $f\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{2}$ is a maximum; since $f''\left(\frac{5\pi}{6}\right) = 3\sqrt{3} > 0$, $f\left(\frac{5\pi}{6}\right) = -\frac{3\sqrt{3}}{2}$ is a minimum.

25 Since $f''(0) = \frac{1}{4} > 0$, $f(0) = 1$ is a minimum.

27 Since $f''\left(\frac{\pi}{4}\right) = -8 < 0$, $f\left(\frac{\pi}{4}\right) = 1$ is a maximum.

29 Since $f''\left(-\frac{11\pi}{6}\right) = f''\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} < 0$, $f\left(-\frac{11\pi}{6}\right) \approx -2.01$ and $f\left(\frac{\pi}{6}\right) \approx 1.13$ are local maxima. Since $f''\left(-\frac{7\pi}{6}\right) = f''\left(\frac{5\pi}{6}\right) = \frac{\sqrt{3}}{2} > 0$, $f\left(-\frac{7\pi}{6}\right) \approx -2.70$ and $f\left(\frac{5\pi}{6}\right) \approx 0.44$ are local minima.

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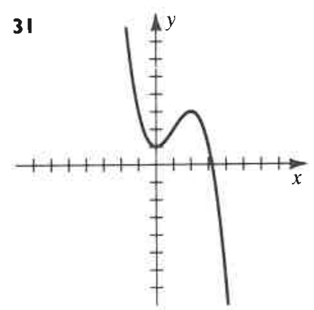

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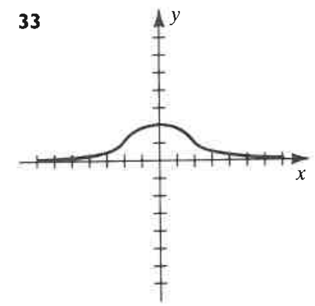

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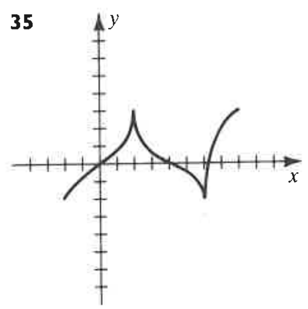

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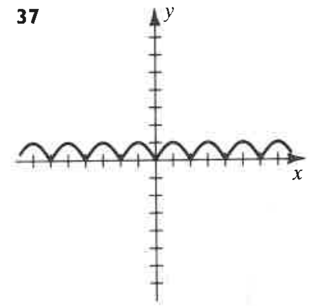
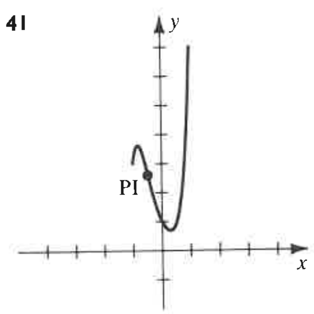


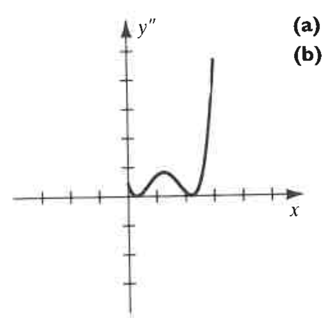
39 If $f(x) = ax^2 + bx + c$, then $f''(x) = 2a$, which does not change sign. Thus, there is no point of inflection.
(a) CU if $a > 0$. **(b)** CD if $a < 0$.

41



(a) CU on $(-0.48, 1)$; CD on $(-1, -0.48)$
(b) -0.48

43



(a) CU on $(0, 3)$
(b) No PI on $(0, 3)$

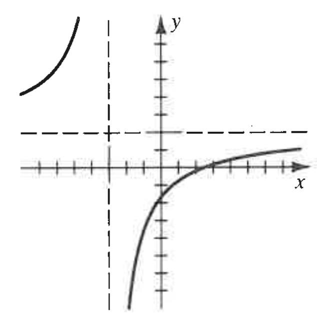
45 **(a)** Min: $f(2.42) \approx -0.90$
(b) PI: $(0, 17)$, $\left(\frac{3}{2}, \frac{95}{16}\right)$; CU on $[-10, 0)$ and $\left(\frac{3}{2}, 10\right]$; CD on $\left(0, \frac{3}{2}\right)$

47 **(a)** Max: $f(0.21) \approx 1$, $f(1.37) \approx 1$, $f(3.50) \approx -0.17$, $f(5.63) \approx 1$;
min: $f(0.73) \approx -1$, $f(2.32) \approx -1$, $f(4.68) \approx -1$
(b) PI: $(0.45, 0.05)$, $(1.01, -0.08)$, $(1.73, 0.16)$, $(2.75, -0.67)$, $(4.25, -0.67)$, $(5.27, 0.16)$, $(5.99, -0.08)$;
CU on $(0.45, 1.01)$, $(1.73, 2.75)$, $(4.25, 5.27)$, $(5.99, 6]$;
CD on $[0, 0.45)$, $(1.01, 1.73)$, $(2.75, 4.25)$, $(5.27, 5.99)$

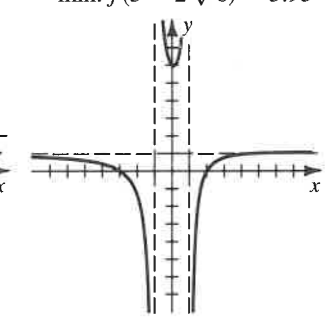
49 **(a)** Max: $f(-1.48) \approx 15.48$, $f(0.67) \approx 1.32$;
min: $f(0) = 0$, $f(3.21) \approx -93.73$
(b) PI: $(-0.96, 9.26)$, $(0.35, 0.67)$, $(2.41, -57.38)$;
CU on $(-0.96, 0.35)$ and $(2.41, 6]$;
CD on $[-4, -0.96)$ and $(0.35, 2.41)$

Exercises 3.5

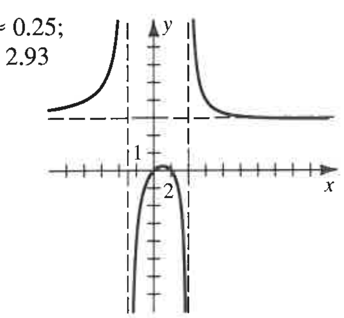
1 No extrema



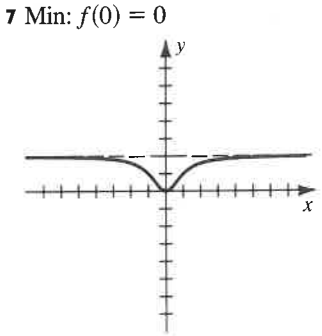
3 Max: $f(5 + 2\sqrt{6}) \approx 1.05$;
min: $f(5 - 2\sqrt{6}) \approx 5.95$



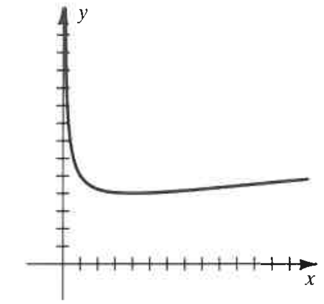
5 Max: $f(12 - 2\sqrt{30}) \approx 0.25$;
min: $f(12 + 2\sqrt{30}) \approx 2.93$



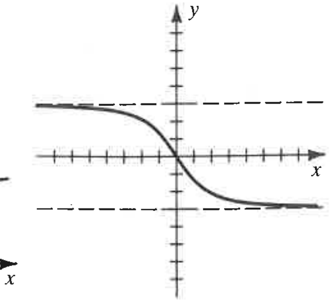
7 Min: $f(0) = 0$



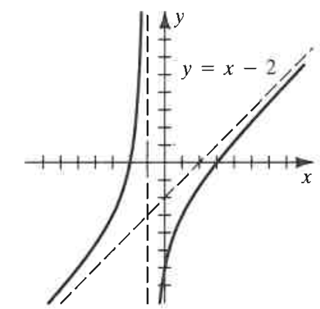
9 Min: $f(4) = 4$



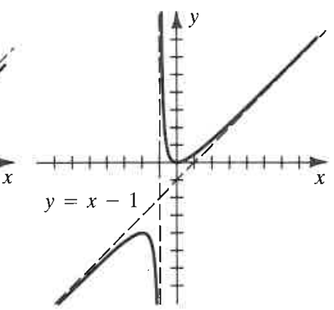
11 No extrema



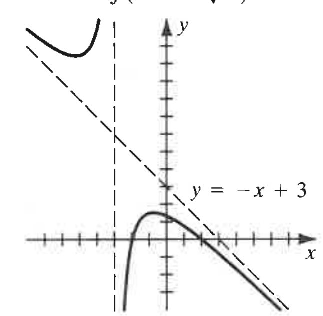
13 No extrema



15 Max: $f(-2) = -4$;
min: $f(0) = 0$



17 Max: $f(-3 + \sqrt{5}) \approx 1.53$; min: $f(-3 - \sqrt{5}) \approx 10.47$



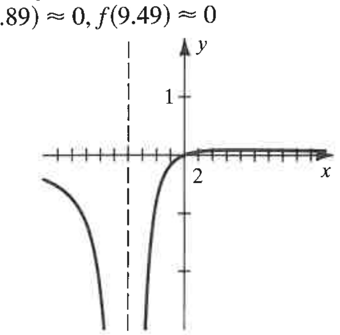
19 Max: $f(0.36) \approx 3.63$, $f(2.54) \approx 1.42$;
min: $f(1.46) \approx -1.42$, $f(3.64) \approx -3.63$

21 Max: $f(\pm 0.44) \approx -4.49$;
min: $f(0) = -4.8$, $f(\pm 1.25) \approx -11.37$

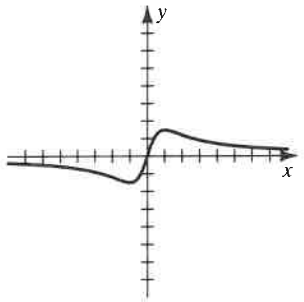
23 Max: $f(3.86) \approx 17.14$; min: 0 for $x \geq 6$ or $x < 1$

25 Max: $f(-0.10) \approx -65.10$, $f(6.77) \approx 96.58$;
min: $f(-2.37) \approx 0$, $f(2.89) \approx 0$, $f(9.49) \approx 0$

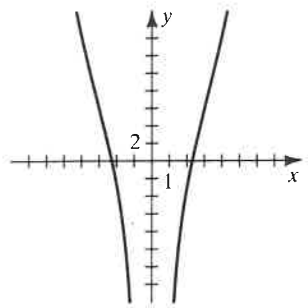
27 Max: $f(8) = \frac{3}{32}$;
PI: $\left(16, \frac{1}{12}\right)$



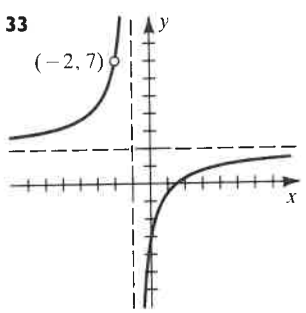
29 Max: $f(1) = \frac{3}{2}$;
min: $f(-1) = -\frac{3}{2}$;
PI: $\left(\pm\sqrt{3}, \pm\frac{3}{4}\sqrt{3}\right)$, $(0, 0)$



31 No extrema;
PI: $(\pm 3, 6)$



33



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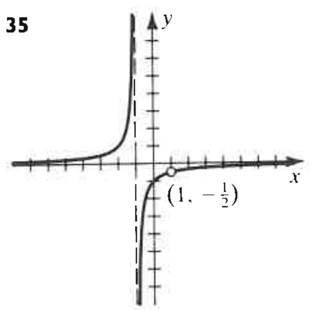
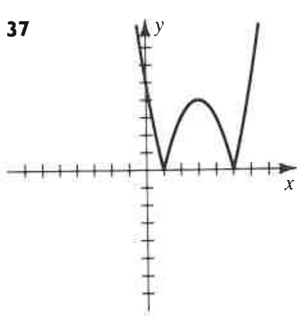


37



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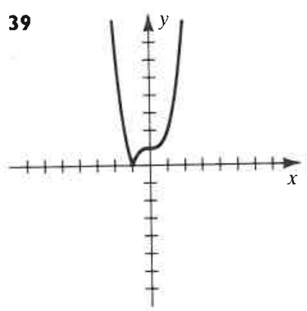

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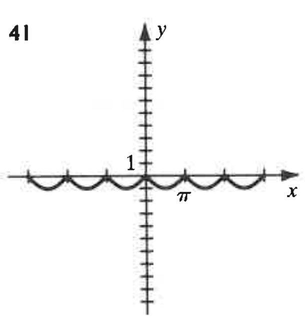

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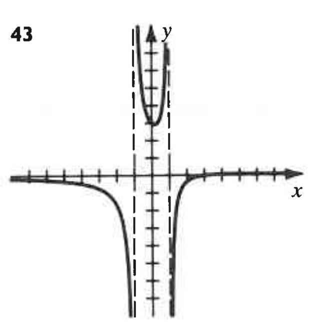
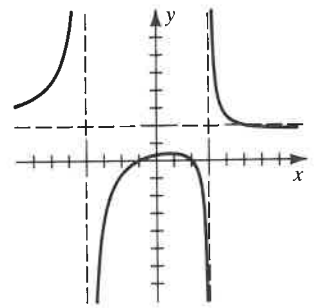
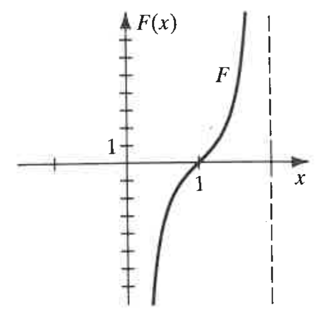


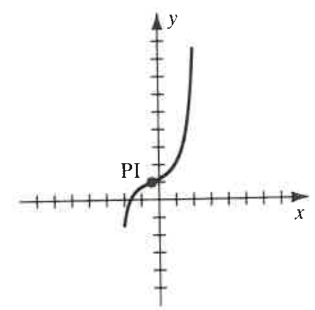
45



47 (b)



49



(a) CU on $(-0.43, 2)$; CD on $(-2, -0.43)$
(b) -0.43

53 (b) $m + 3n - 2$ (before simplification)

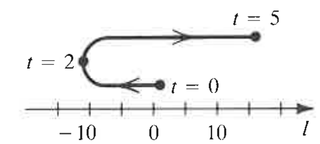
Exercises 3.6

1 225 **3** 71 **5** 800
7 Side of base = 2 ft; height = 1 ft
9 Radius of base = height = $\frac{1}{\sqrt[3]{\pi}}$
11 $x = 166\frac{2}{3}$ ft; $y = 125$ ft
13 Approximately 2:23:05 P.M. **15** $5\sqrt{5} \approx 11.18$ ft
17 Length = $2\sqrt[3]{300} \approx 13.38$ ft;
width = $\frac{3}{2}\sqrt[3]{300} \approx 10.04$ ft;
height = $\sqrt[3]{300} \approx 6.69$ ft
21 55 **23** Radius = $\frac{1}{2}\sqrt[3]{15}$; length of cylinder = $2\sqrt[3]{15}$
25 Length of base = $\sqrt{2}a$; height = $\frac{1}{2}\sqrt{2}a$ **27** $\frac{32}{81}\pi a^3$
29 (1, 2) **31** Width = $\frac{2}{\sqrt{3}}a$; depth = $\frac{2\sqrt{2}}{\sqrt{3}}a$ **33** 500
35 (a) Use $\frac{36\sqrt{3}}{2 + \sqrt{3}} \approx 16.71$ cm for the rectangle.
(b) Use all the wire for the rectangle.
37 Width = $\frac{12}{6 - \sqrt{3}} \approx 2.81$ ft;
height = $\frac{18 - 6\sqrt{3}}{6 - \sqrt{3}} \approx 1.78$ ft

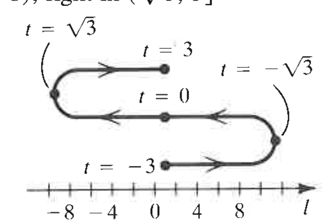
41 37 **43** 18 in., 18 in., 36 in.
45 $\frac{4}{1 + \sqrt[4]{\frac{1}{2}}} \approx 2.17$ mi from A
49 (c) $4\sqrt{30} \approx 21.9$ mi/hr
51 60° **53** $2\pi\left(1 - \frac{1}{3}\sqrt{6}\right)$ radians $\approx 66.06°$
55 $\tan\theta = \frac{\sqrt{2}}{2}$; $\theta \approx 35.3°$
59 $\tan\theta = \sqrt[3]{\frac{4}{3}}$; $\theta \approx 47.74°$; $L = \frac{4}{\sin\theta} + \frac{3}{\cos\theta} \approx 9.87$ ft
61 (b) $\cos\theta = \frac{2}{3}$; $\theta \approx 48.2°$

Exercises 3.7

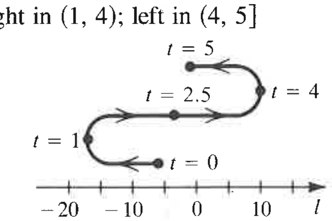
1 $v(t) = 6(t - 2)$; $a(t) = 6$; left in $[0, 2)$; right in $(2, 5]$



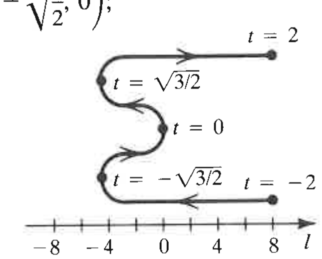
3 $v(t) = 3(t^2 - 3)$; $a(t) = 6t$; right in $[-3, -\sqrt{3})$; left in $(-\sqrt{3}, \sqrt{3})$; right in $(\sqrt{3}, 3]$



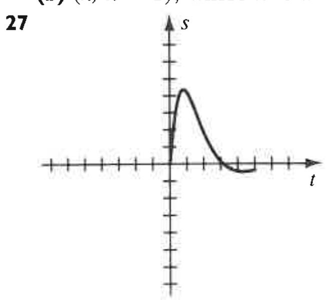
5 $v(t) = -6(t - 1)(t - 4)$; $a(t) = -6(2t - 5)$; left in $[0, 1)$; right in $(1, 4)$; left in $(4, 5]$



7 $v(t) = 4t(2t^2 - 3)$; $a(t) = 12(2t^2 - 1)$; left in $\left[-2, -\sqrt{\frac{3}{2}}\right)$; right in $\left(-\sqrt{\frac{3}{2}}, 0\right)$;
left in $\left(0, \sqrt{\frac{3}{2}}\right)$;
right in $\left(\sqrt{\frac{3}{2}}, 2\right]$



9 (a) 30 ft/sec **(b)** 2.8 sec
11 (a) $v(t) = 16(9 - 2t)$; $a(t) = -32$ **(b)** 324 ft **(c)** 9 sec
13 5; 8; $\frac{1}{8}$ **15** 6; 3; $\frac{1}{3}$ **17** $79{,}200\pi$; $3600\sqrt{2\pi}$
19 (a) $y = 4.5\sin\left[\frac{\pi}{6}(t - 10)\right] + 7.5$
$= 4.5\sin\left(\frac{\pi}{6}t - \frac{5\pi}{3}\right) + 7.5$
(b) 1.178 ft/hr
21 (a) In in./sec: 0, $-\pi$, 0, π, 0
(b) $(n, n + 1)$, where n is an odd positive integer
27



t	0	1	2	3	4	5
s	0	4.21	1.82	0.14	−0.45	−0.37
v	10	−1.51	−2.29	−1.07	−0.18	0.25
a	0	−5.40	1.11	1.12	0.66	0.20

29 (a) 1600 ft **(b)** No, speed is about 218 mi/hr on impact. **(c)** 1089 ft
31 (a) 3 sec **(b)** 40 ft/sec
33 For $a > 0$ and measured in ft/sec², $v_0 = v_1 + \frac{15}{22}\sqrt{2as}$.

Exercises 3.8

1 (a) 806
(b) $c(x) = \frac{800}{x} + 0.04 + 0.0002x$;
$C'(x) = 0.04 + 0.0004x$; $c(100) = 8.06$;
$C'(100) = 0.08$
3 (a) 11,250
(b) $c(x) = \frac{250}{x} + 100 + 0.001x^2$;
$C'(x) = 100 + 0.003x^2$; $c(100) = 112.50$;
$C'(100) = 130$
5 $C'(5) = \$46$; $C(6) - C(5) \approx \$46.67$
7 (a) -0.1 **(b)** $50x - 0.1x^2$ **(c)** $48x - 0.1x^2 - 10$ **(d)** $48 - 0.2x$ **(e)** 5750 **(f)** 2
9 (a) $1800x - 2x^2$ **(b)** $1799x - 2.01x^2 - 1000$ **(c)** 100 **(d)** \$158,800

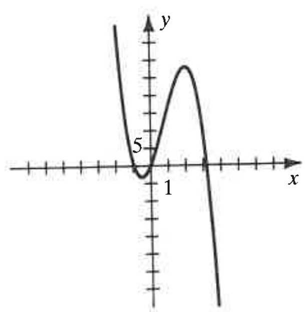
11 (a) 3990 mills **(b)** \$15,420.10

13 The stable point occurs at $\left(\frac{m}{n}, \frac{a}{b}\right)$.

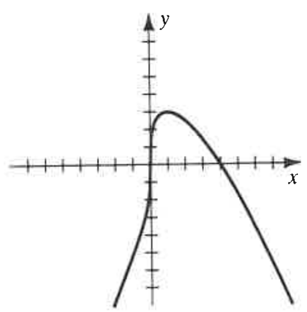
Chapter 3 Review Exercises

1 Max: $f(3) = 1$; min: $f(6) = -8$ **3** $-2, -1, \frac{1}{3}$

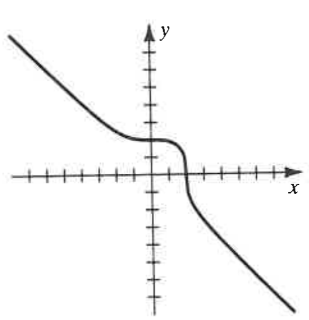
5 Max: $f(2) = 28$; min: $f\left(-\frac{1}{2}\right) = -\frac{13}{4}$; increasing on $\left[-\frac{1}{2}, 2\right]$; decreasing on $\left(-\infty, -\frac{1}{2}\right]$ and $[2, \infty)$



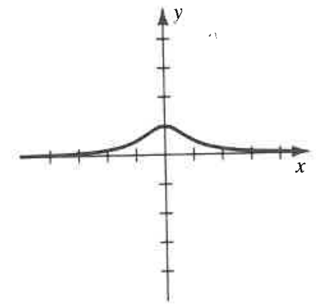
7 Max: $f(1) = 3$; increasing on $(-\infty, 1]$; decreasing on $[1, \infty)$



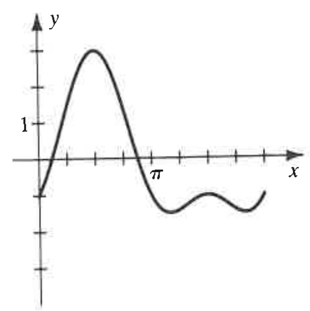
9 Since $f''(0) = 0$ and $f''(2)$ is undefined, use the first derivative test to show that there are no extrema; CU on $(-\infty, 0)$ and $(2, \infty)$; CD on $(0, 2)$; x-coordinates of PI are 0 and 2.



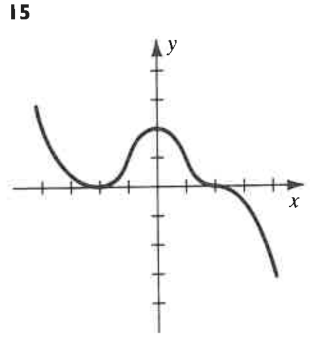
11 Since $f''(0) = -2 < 0$, $f(0) = 1$ is a maximum; CU on $\left(-\infty, -\frac{1}{3}\sqrt{3}\right)$ and $\left(\frac{1}{3}\sqrt{3}, \infty\right)$; CD on $\left(-\frac{1}{3}\sqrt{3}, \frac{1}{3}\sqrt{3}\right)$; x-coordinates of PI are $\pm\frac{1}{3}\sqrt{3}$.



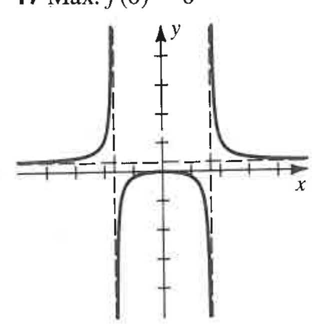
13 Max: $f\left(\frac{\pi}{2}\right) = 3$ and $f\left(\frac{3\pi}{2}\right) = -1$; min: $f\left(\frac{7\pi}{6}\right) = f\left(\frac{11\pi}{6}\right) = -\frac{3}{2}$



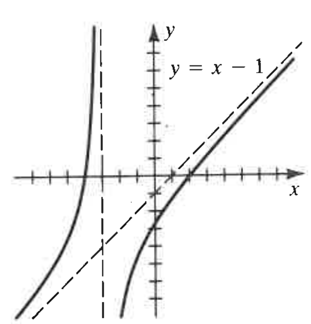
15



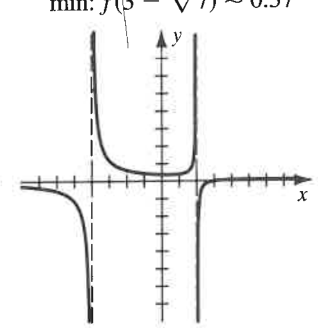
17 Max: $f(0) = 0$



19 No extrema



21 Max: $f(3 + \sqrt{7}) \approx 0.08$; min: $f(3 - \sqrt{7}) \approx 0.37$



23 $\frac{\sqrt{61} - 1}{3}$ **25** 125 yd by 250 yd **27** $\frac{\pi}{2}$

29 Radius of semicircle is $\frac{1}{8\pi}$ mi, length of rectangle is $\frac{1}{8}$ mi.

31 (a) Use all the wire for the circle.

(b) Use length $\frac{5\pi}{4 + \pi} \approx 2.2$ ft for the circle and the remainder for the square.

33 $v(t) = \frac{3(1 - t^2)}{(t^2 + 1)^2}$; $a(t) = \frac{6t(t^2 - 3)}{(t^2 + 1)^3}$; left in $[-2, -1)$; right in $(-1, 1)$; left in $(1, 2]$

35 $C'(100) = 116$; $C(101) - C(100) = 116.11$

37 (a) $18x$ **(b)** $-0.02x^2 + 12x - 500$ **(c)** 300 **(d)** \$1300

39 98 ft/sec² **41** 2.27 **43** ± 0.79

45 Min: $f(1.5345) \approx -10.2624$; PI: none

47 Max: $f(0.3666) \approx 0.3240$; min: $f(0.4780) \approx 0$, $f(0.2527) \approx 0$; PI: $(0.4780, 0)$ and $(0.2527, 0)$

49 Max: $f(1.0810) \approx 2.2948$; min: $f(0.5643) \approx 2.1902$; PI: $(-0.8281, 5.5559)$ and $(0.8281, 2.2434)$

CHAPTER ▪ 4

Exercises 4.1

1 $2x^2 + 3x + C$ **3** $3t^3 - 2t^2 + 3t + C$

5 $-\frac{1}{2z^2} + \frac{3}{z} + C$ **7** $2u^{3/2} + 2u^{1/2} + C$

9 $\frac{8}{9}v^{9/4} + \frac{24}{5}v^{5/4} - v^{-3} + C$ **11** $3x^3 - 3x^2 + x + C$

13 $\frac{2}{3}x^3 + \frac{3}{2}x^2 + C$ **15** $\frac{24}{5}x^{5/3} - \frac{15}{2}x^{2/3} + C$

17 $\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$ **19** $-t^{-1} - 2t^{-3} - \frac{9}{5}t^{-5} + C$

21 $\frac{3}{4}\sin u + C$ **23** $-7\cos x + C$

25 $\frac{2}{3}t^{3/2} + \sin t + C$ **27** $\tan t + C$ **29** $-\cot v + C$

31 $\sec w + C$ **33** $-\csc z + C$ **35** $\sqrt{x^2 + 4} + C$

37 $\sin\sqrt[3]{x} + C$ **39** $x^3\sqrt{x - 4}$

43 $a^2x + C$ **45** $\frac{1}{2}at^2 + bt + C$ **47** $(a + b)u + C$

49 $f(x) = 4x^3 - 3x^2 + x + 3$ **51** $y = \frac{8}{3}x^{3/2} - \frac{1}{3}$

53 $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 8x + \frac{65}{6}$

55 $y = -3\sin x + 4\cos x + 5x + 3$ **57** $t^2 - t^3 - 5t + 4$

59 (a) $s(t) = -16t^2 + 1600t$ **(b)** $s(50) = 40{,}000$ ft

61 (a) $s(t) = -16t^2 - 16t + 96$ **(b)** $t = 2$ sec **(c)** -80 ft/sec

63 Solve the differential equation $s''(t) = -g$ for $s(t)$.

65 10 ft/sec² **67** 19.62

69 $C(x) = 20x - 0.0075x^2 + 5.0075$; $C(50) \approx \$986.26$

71 $10x^4 + 4x^3 + 27x^2 - 10x + 4$; $\frac{1}{3}x^6 + \frac{1}{5}x^5 + \frac{9}{4}x^4 - \frac{5}{3}x^3 + 2x^2 + 10x + C$

73 $e^{3x}[(3x^2 + 2x)\cos(4x) - 4x^2\sin(4x)]$; $\frac{1}{15{,}625}e^{3x}[(1875x^2 + 350x - 234)\cos(4x) + 4(625x^2 - 300x + 22)\sin(4x)] + C$

75 $\frac{-(4t^3 - 27t^2 - 30t + 31)}{(2t^3 + 3t^2 - 5t - 6)^2}$; $-\ln(2t - 3)^{1/5} - \ln(t + 2) + \frac{6}{5}\ln(t + 1) + C$

77 (b) Each pair of functions differs only by a constant.

Exercises 4.2

1 $\frac{1}{44}(2x^2 + 3)^{11} + C$ **3** $\frac{1}{12}(3x^3 + 7)^{4/3} + C$

5 $\frac{1}{2}(1 + \sqrt{x})^4 + C$ **7** $\frac{2}{3}\sin\sqrt{x^3} + C$

9 $\frac{2}{9}(3x - 2)^{3/2} + C$ **11** $\frac{3}{32}(8t + 5)^{4/3} + C$

13 $\frac{1}{15}(3z + 1)^5 + C$ **15** $\frac{2}{9}(v^3 - 1)^{3/2} + C$

17 $-\frac{3}{8}(1 - 2x^2)^{2/3} + C$ **19** $\frac{1}{5}s^5 + \frac{2}{3}s^3 + s + C$

21 $\frac{2}{5}(\sqrt{x} + 3)^5 + C$ **23** $-\frac{1}{4(t^2 - 4t + 3)^2} + C$

25 $-\frac{3}{4}\cos 4x + C$ **27** $\frac{1}{4}\sin(4x - 3) + C$

29 $-\frac{1}{2}\cos(v^2) + C$ **31** $\frac{1}{4}(\sin 3x)^{4/3} + C$

33 $x - \frac{1}{2}\cos 2x + C$ **35** $-\cos x - \cos^2 x - \frac{1}{3}\cos^3 x + C$

37 $\frac{1}{3\cos^3 x} + C$ **39** $\frac{1}{1 - \sin t} + C$ **41** $\frac{1}{3}\tan(3x - 4) + C$

43 $\frac{1}{6}\sec^2 3x + C$ **45** $-\frac{1}{5}\cot 5x + C$

47 $-\frac{1}{2}\csc(x^2) + C$ **49** $f(x) = \frac{1}{4}(3x + 2)^{4/3} + 5$

51 $f(x) = 3\sin x - 4\cos 2x + x + 2$

53 (a) $\frac{1}{3}(x + 4)^3 + C_1$

(b) $\frac{1}{3}x^3 + 4x^2 + 16x + C_2$; $C_2 = C_1 + \frac{64}{3}$

55 (a) $\frac{2}{3}(\sqrt{x} + 3)^3 + C_1$

(b) $\frac{2}{3}x^{3/2} + 6x + 18x^{1/2} + C_2$; $C_2 = C_1 + 18$

59 474,592 ft³ **61 (a)** $\frac{dV}{dt} = 0.6 \sin\left(\frac{2\pi}{5}t\right)$ **(b)** $\frac{3}{\pi} \approx 0.95$ L

63 *Hint:* (i) Let $u = \sin x$. (ii) Let $u = \cos x$. (iii) Use the double angle formula for the sine. The three answers differ by constants.

Exercises 4.3

1 34 **3** 40 **5** 10 **7** 500

9 $\frac{1}{3}n(n^2 + 6n + 20)$ **11** $\frac{1}{12}n(3n^3 + 14n^2 + 9n + 46)$

Exer. 13–18: Answers are not unique.

13 $\sum_{k=1}^{5}(4k-3)$ **15** $\sum_{k=1}^{4}\frac{k}{3k-1}$ **17** $1 + \sum_{k=1}^{n}(-1)^k\frac{x^{2k}}{2k}$

19 111,142.3744 **21** 7.4855

23 0.9441 **25** 21,781,332

27 (a) 10 **(b)** 14

29 (a) $\frac{35}{4}$ **(b)** $\frac{51}{4}$ **31 (a)** 1.04 **(b)** 1.19

Exer. 33–38: Answers for (a) and (b) are the same.

33 28 **35** 18 **37** 6 **39 (a)** 20 **(b)** $\frac{1}{4}(b^4 - a^4)$

Exercises 4.4

1 (a) 1.1, 1.5, 1.1, 0.4, 0.9 **(b)** 1.5
3 (a) 0.3, 1.7, 1.4, 0.5, 0.1 **(b)** 1.7
5 (a) 30 **(b)** 42 **(c)** 36
7 (a) 15.127 **(b)** 15.283 **(c)** 15.3975
9 (a) 141 **(b)** 551 **(c)** 307
11 (a) 292.5 **(b)** 348.5 **(c)** 319.75
13 (a) 0.2668 **(b)** 0.2962 **(c)** 0.2813

15 $\int_{-1}^{2}(3x^2 - 2x + 5)\,dx$ **17** $\int_{0}^{4} 2\pi x(1 + x^3)\,dx$

19 $-\frac{14}{3}$ **21** $\frac{14}{3}$ **23** $-\frac{14}{3}$

25 $\int_{0}^{4}\left(-\frac{5}{4}x + 5\right)dx$ **27** $\int_{-1}^{5}\sqrt{9 - (x-2)^2}\,dx$

29 36 **31** 25 **33** 2.5 **35** $\frac{9\pi}{4}$ **37** $12 + 2\pi$

Exercises 4.5

1 30 **3** −12 **5** 2 **7** 78 **9** $-\frac{291}{2}$

11 Use Corollary (4.27). **13** Use Theorem (4.26).

15 Use Theorem (4.26). **17** $\int_{-3}^{1} f(x)\,dx$

19 $\int_{e}^{d} f(x)\,dx$ **21** $\int_{h}^{c+h} f(x)\,dx$ **23 (a)** $\sqrt{3}$ **(b)** 9

25 (a) −1 **(b)** 2 **27 (a)** 3 **(b)** 6

29 (a) $\sqrt[3]{\frac{15}{4}}$ **(b)** 14

31 1.426 **33** Use (4.22) and (4.23)(i).

Exercises 4.6

1 −18 **3** $\frac{265}{2}$ **5** 5 **7** $\frac{31}{32}$ **9** $\frac{20}{3}$ **11** $\frac{352}{5}$

13 $\frac{13}{3}$ **15** $-\frac{7}{2}$ **17** 0 **19** $\frac{10}{3}$ **21** $\frac{53}{2}$ **23** $\frac{14}{3}$

25 0 **27** $\frac{1}{3}$ **29** $\frac{5}{36}$ **31** $\frac{3}{2}(\sqrt{3} - 1) \approx 1.10$

33 $1 - \sqrt{2} \approx -0.41$ **35** 0

37 No, $\sec^2 x$ is not continuous on $[0, \pi]$.

39 Yes, since $\int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx = \int_{-1}^{1} f(x)\,dx$.

41 (a) $\sqrt{3}$ **(b)** $\frac{1}{2}$ **43 (a)** $\frac{544}{225}$ **(b)** $\frac{38}{15}$

45 0 **47** $\frac{1}{x+1}$ **51 (a)** $\frac{6}{7}cd^{1/6}$

55 *Hint:* Use Part I of the fundamental theorem of calculus (4.30) and the chain rule.

57 $\frac{4x^7}{\sqrt{x^{12} + 2}}$ **59** $3x^2(x^9 + 1)^{10} - 3(27x^3 + 1)^{10}$

Exercises 4.7

1 $L_6 = 10.95$; $R_6 = 11.95$; $M_3 = 11.1$; $T_6 = 11.45$; $S_3 = 11\frac{1}{3}$
3 $L_8 = 12.33375$; $R_8 = 13.60875$; $M_4 = 12.6975$; $T_8 = 12.97125$; $S_4 = 12.88$
5 (a) $L_8 = 1.1501$; $R_8 = 1.2597$ **(b)** 1.2049
7 (a) $L_3 = 0.84$; $L_6 = 0.9$; $L_{12} = 0.93$
(b) 0.96; $E_3 = 0.12$; $E_6 = 0.06$; $E_{12} = 0.03$
(c) The error is reduced by $\frac{1}{2}$ when n doubles.
9 (a) $M_2 = 144$; $M_4 = 153$; $M_8 = 155.25$
(b) 156; $E_2 = 12$; $E_4 = 3$; $E_8 = 0.75$
(c) The error is reduced by $\frac{1}{4}$ when n doubles.
11 (a) $T_2 = 180$; $T_4 = 162$; $T_8 = 157.5$
(b) 156; $E_2 = -24$; $E_4 = -6$; $E_8 = -1.5$
(c) The error is reduced by $\frac{1}{4}$ when n doubles.
13 (a) $S_2 = S_4 = S_8 = 156$
(b) 156; $E_2 = E_4 = E_8 = 0$
(c) Simpson's rule is exact for all n.
15 (a) $T_5 \approx 6.249806$; $T_{10} \approx 6.234926$; $T_{20} \approx 6.231201$; $T_{40} \approx 6.230270$
(b) At least two decimal places
17 (a) $S_2 \approx 2.3987529621$; $S_6 \approx S_{18} \approx S_{54} \approx 2.4039394306$
(b) At least ten decimal places
19 (a) 0.26 **(b)** 4.2×10^{-5}
21 (a) 0.125 **(b)** 6.5×10^{-4}
23 (a) 3,386,880 **(b)** 642 **(c)** 10

25 (a) 25 **(b)** 3 **(c)** 1 **29 (a)** 127.5 **(b)** 131.7
31 0.174 m/sec **33** 0.28 **35** 1.48

Chapter 4 Review Exercises

1 $-\frac{8}{x} + \frac{2}{x^2} - \frac{5}{3x^3} + C$ **3** $100x + C$ **5** $\frac{1}{16}(2x + 1)^8 + C$

7 $-\frac{1}{16}(1 - 2x^2)^4 + C$ **9** $-\frac{2}{1 + \sqrt{x}} + C$

11 $3x - x^2 - \frac{5}{4}x^4 + C$ **13** $\frac{1}{6}(4x^2 + 2x - 7)^3 + C$

15 $-\frac{1}{x^2} - x^3 + C$ **17** $\frac{3}{5}$ **19** $\frac{1}{6}$ **21** $\sqrt{8} - \sqrt{3} \approx 1.10$

23 $\frac{52}{9}$ **25** $-\frac{37}{6}$ **27** $8\sqrt{3} + 16 \approx 29.86$

29 $\frac{1}{5}\cos(3 - 5x) + C$ **31** $\frac{1}{15}\sin^5 3x + C$

33 $-\frac{1}{6\sin^2 3x} + C$ **35** $\frac{2}{15}(16\sqrt{2} - 3\sqrt{3}) \approx 2.32$ **37** $\frac{1}{6}$

39 $\sqrt[5]{x^4 + 2x^2 + 1} + C$ **41** 0 **43** $y = x^3 - 2x^2 + x + 2$

45 $\frac{135}{4}$ **47** Use Corollary (4.27). **49** $\int_{a}^{e} f(x)\,dx$

51 (a) $-16t^2 - 30t + 900$ **(b)** −190 ft/sec
(c) $\frac{15}{16}(-1 + \sqrt{65}) \approx 6.6$ sec

53 $\int_{-2}^{3}\sqrt{1 + 3x^2}\,dx$ **55** $M_5 \approx 0.824279$; $M_{10} \approx 0.8092539$

57 $S_4 \approx 11.105304$; $S_8 \approx 11.105302$ **59** 81.625°F

CHAPTER 5

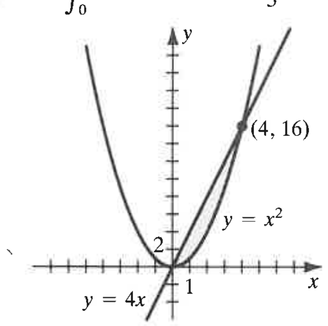
Exercises 5.1

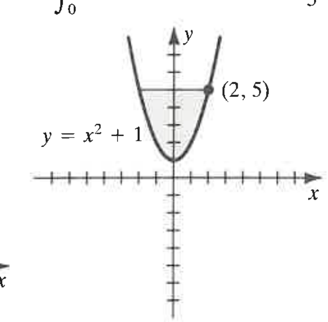
Exer. 1–4: Answers are not unique.

1 $\int_{-2}^{2}[(x^2 + 1) - (x - 2)]\,dx$

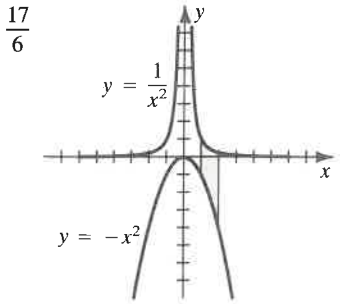
3 $\int_{-2}^{1}[(-3y^2 + 4) - y^3]\,dy$

5 $\int_{0}^{4}(4x - x^2)\,dx = \frac{32}{3}$ **7** $2\int_{0}^{2}[5 - (x^2 + 1)]\,dx = \frac{32}{3}$

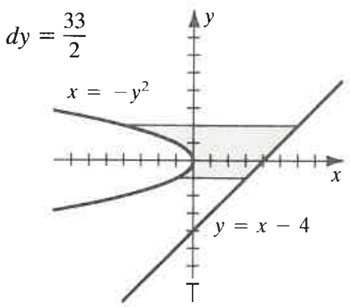




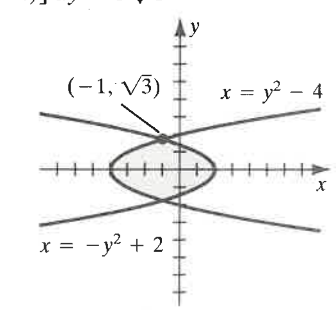
9 $\int_{1}^{2}\left[\frac{1}{x^2} - (-x^2)\right]dx = \frac{17}{6}$



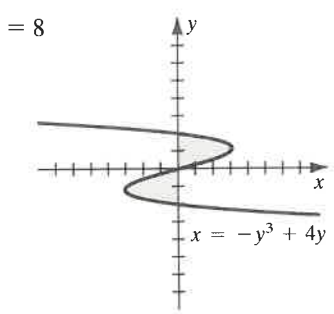
11 $\int_{-1}^{2}[(4 + y) - (-y^2)]\,dy = \frac{33}{2}$



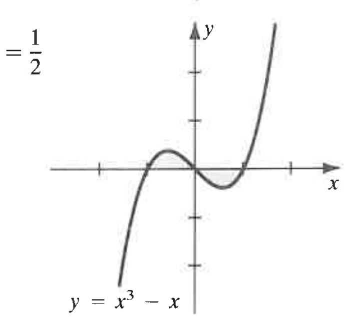
13 $2\int_{0}^{\sqrt{3}}[(2 - y^2) - (y^2 - 4)]\,dy = 8\sqrt{3}$



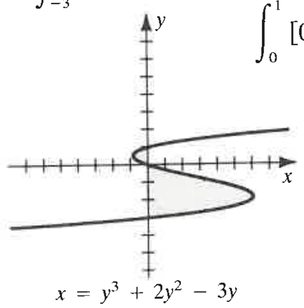
15 $2\int_{0}^{2}[(4y - y^3) - 0]\,dy = 8$



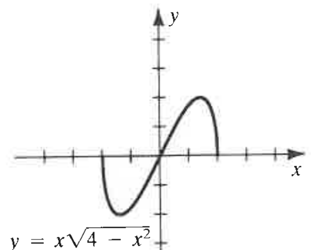
17 $2\int_{0}^{1}[0 - (x^3 - x)]\,dx = \frac{1}{2}$



19 $\int_{-3}^{0} [(y^3 + 2y^2 - 3y) - 0]\, dy + \int_{0}^{1} [0 - (y^3 + 2y^2 - 3y)]\, dy = \frac{71}{6}$



21 $\int_{0}^{2} x\sqrt{4 - x^2}\, dx = \frac{16}{3}$



23 $3 + \frac{3}{2}\sqrt{3} \approx 5.74$

25 (a) $\int_{0}^{1} (3x - x)\, dx + \int_{1}^{2} [(4 - x) - x]\, dx$

(b) $\int_{0}^{2} \left(y - \frac{1}{3}y\right) dy + \int_{2}^{3} \left[(4 - y) - \frac{1}{3}y\right] dy$

27 (a) $\int_{1}^{4} [\sqrt{x} - (-x)]\, dx$

(b) $\int_{-4}^{-1} [4 - (-y)]\, dy + \int_{-1}^{1} (4 - 1)\, dy + \int_{1}^{2} (4 - y^2)\, dy$

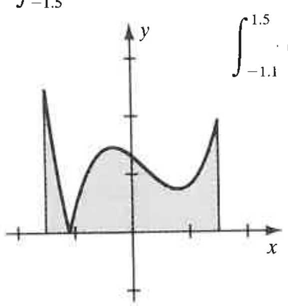
29 (a) $\int_{-6}^{-1} [(x + 3) - (-\sqrt{3 - x})]\, dx + 2\int_{-1}^{3} \sqrt{3 - x}\, dx$

(b) $\int_{-3}^{2} [(3 - y^2) - (y - 3)]\, dy$

31 9 **33** 12 **35** $4\sqrt{2}$

37 $\int_{0}^{1} (x^2 - 6x + 5)\, dx + \int_{1}^{5} -(x^2 - 6x + 5)\, dx + \int_{5}^{7} (x^2 - 6x + 5)\, dx$

41 $\int_{-1.5}^{-1.1} -(x^3 - 0.7x^2 - 0.8x + 1.3)\, dx + \int_{-1.1}^{1.5} (x^3 - 0.7x^2 - 0.8x + 1.3)\, dx$



43 (a) $(a, 0.9052)$, $(b, 5.3623)$, $a \approx 0.0819$, $b \approx 2.8754$

(b) $\int_{a}^{b} [\sqrt{10x} - (x^3 - 2x^2 - x + 1)]\, dx$ **(c)** 10.3259

45 (a) $(\pm a, -8.0061)$, $a \approx 3.4632$

(b) $\int_{-a}^{a} [50 \cos(0.5x) - (x^2 - 20)]\, dx$ **(c)** 308.2566

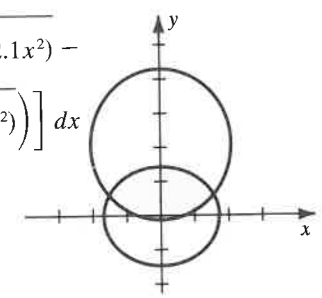
47 (a) $\int_{-5}^{5} [\sqrt{25 - x^2} - (\sqrt{29 - x^2} - 2)]\, dx$ **(b)** 14.7515

49 (a) $\int_{0}^{\pi} [\sin x - \sin(\sin x)]\, dx$ **(b)** 0.2135

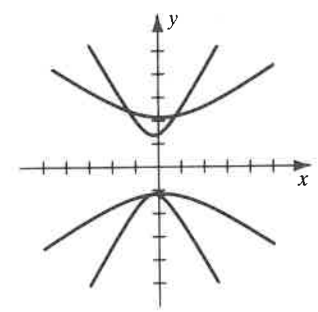
51 (a) $[0, 1]$ **(b)** $\frac{1}{6}$ **53 (a)** $[0, 1]$ **(b)** 2

55 (a) $(\pm 1.540, 0.618)$

(b) $2\int_{0}^{1.54} \left[\sqrt{\frac{1}{2.9}(6.09 - 2.1x^2)} - \left(2.1 - \sqrt{\frac{1}{4.3}(21.07 - 4.9x^2)}\right)\right] dx$



57 (a) $(0.741, 2.206)$

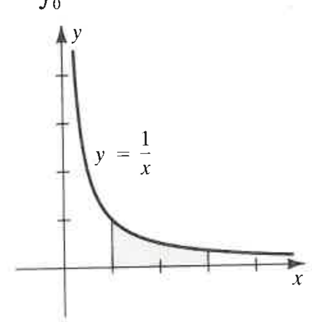


(b) $\int_{0}^{0.74} \left\{0.5 + \sqrt{\frac{1}{5.3}[14.31 + 2.7(x - 0.1)^2]} - [0.1 + \sqrt{1.6 + 3.2(x + 0.2)^2}]\right\} dx$

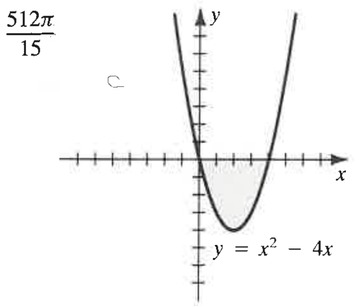
Exercises 5.2

1 $\pi\int_{-1}^{2} \left(\frac{1}{2}x^2 + 2\right)^2 dx$ **3** $2 \cdot \pi\int_{0}^{4} [(\sqrt{25 - y^2})^2 - 3^2]\, dy$

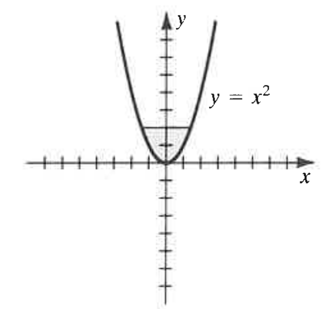
5 $\pi\int_{1}^{3} \left(\frac{1}{x}\right)^2 dx = \frac{2\pi}{3}$

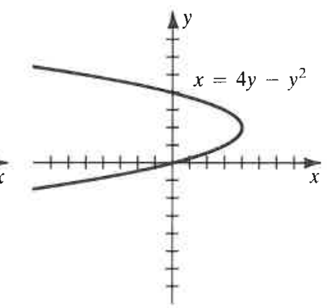


7 $\pi\int_{0}^{4} (x^2 - 4x)^2\, dx = \frac{512\pi}{15}$



9 $\pi\int_{0}^{2} (\sqrt{y})^2\, dy = 2\pi$ **11** $\pi\int_{0}^{4} (4y - y^2)^2\, dy = \frac{512\pi}{15}$

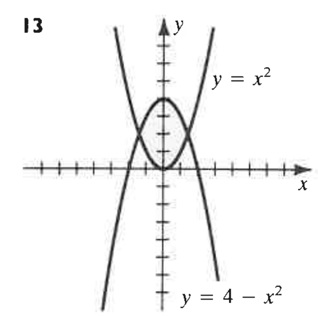




13 $2 \cdot \pi\int_{0}^{\sqrt{2}} [(4 - x^2)^2 - (x^2)^2]\, dx = \frac{64\pi\sqrt{2}}{3}$

15 $\pi\int_{0}^{2} [(4 - x)^2 - (x)^2]\, dx = 16\pi$

13



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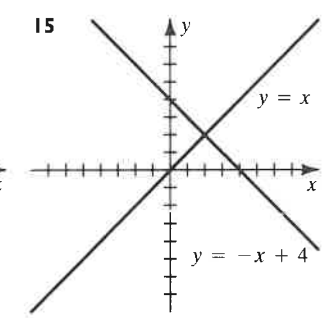
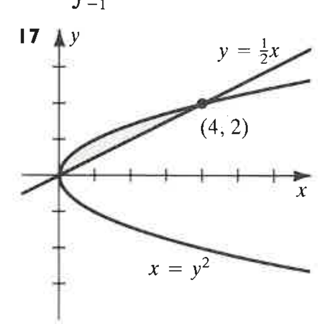


17 $\pi\int_{0}^{2} [(2y)^2 - (y^2)^2]\, dy = \frac{64\pi}{15}$

19 $\pi\int_{-1}^{2} [(y + 2)^2 - (y^2)^2]\, dy = \frac{72\pi}{5}$

17



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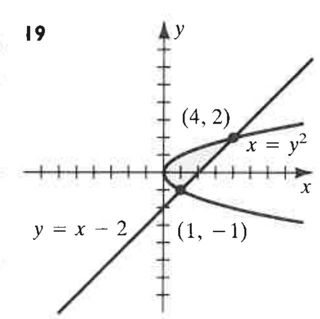
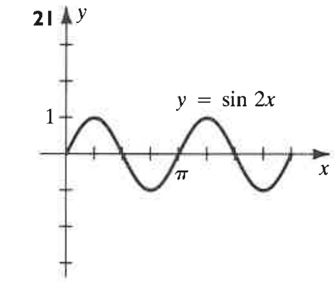


21 $\pi\int_{0}^{\pi} (\sin 2x)^2\, dx = \frac{1}{2}\pi^2$

23 $\pi\int_{0}^{\pi/4} [(\cos x)^2 - (\sin x)^2]\, dx = \frac{\pi}{2}$

21



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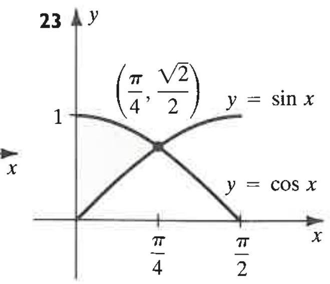

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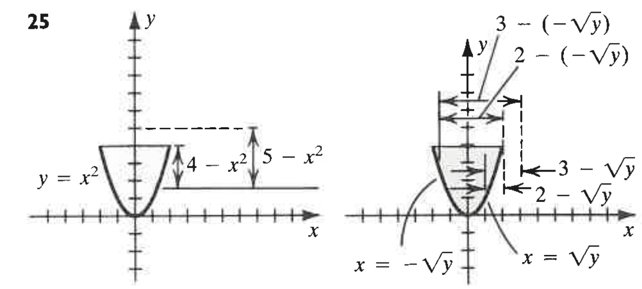


(a) $2 \cdot \pi\int_{0}^{2} (4 - x^2)^2\, dx = \frac{512\pi}{15}$

(b) $2 \cdot \pi\int_{0}^{2} [(5 - x^2)^2 - (5 - 4)^2]\, dx = \frac{832\pi}{15}$

(c) $\pi\int_{0}^{4} \{[2 - (-\sqrt{y})]^2 - [2 - \sqrt{y}]^2\}\, dy = \frac{128\pi}{3}$

(d) $\pi\int_{0}^{4} \{[3 - (-\sqrt{y})]^2 - [3 - \sqrt{y}]^2\}\, dy = 64\pi$

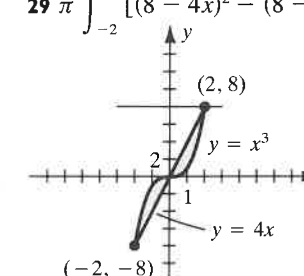
27 (a) $\pi\int_{0}^{4} \left\{\left[\left(-\frac{1}{2}x + 2\right) - (-2)\right]^2 - [0 - (-2)]^2\right\} dx$

(b) $\pi\int_{0}^{4} \left\{(5 - 0)^2 - \left[5 - \left(-\frac{1}{2}x + 2\right)\right]^2\right\} dx$

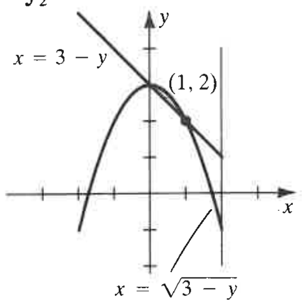
(c) $\pi\int_{0}^{2} \{(7 - 0)^2 - [7 - (-2y + 4)]^2\}\, dy$

(d) $\pi\int_{0}^{2} \{[(-2y + 4) - (-4)]^2 - [0 - (-4)]^2\}\, dy$

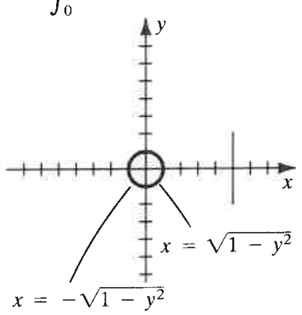
29 $\pi\int_{-2}^{0} [(8 - 4x)^2 - (8 - x^3)^2]\, dx + \pi\int_{0}^{2} [(8 - x^3)^2 - (8 - 4x)^2]\, dx$



31 $\pi\int_2^3 \{[2-(3-y)]^2-[2-\sqrt{3-y}]^2\}\,dy$



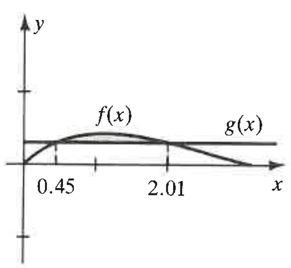
33 $2\cdot\pi\int_0^1 \{[5-(-\sqrt{1-y^2})]^2-[5-\sqrt{1-y^2}]^2\}\,dy$



35 $\pi\int_0^h r^2\,dy=\pi r^2 h$ **37** $\pi\int_0^h\left(\frac{r}{h}x\right)^2dx=\frac{1}{3}\pi r^2h$

39 $\pi\int_0^h\left(\frac{R-r}{h}x+r\right)^2dx=\frac{1}{3}\pi h(R^2+Rr+r^2)$ **41** $\frac{63\pi}{2}$

43 **(a)** 0.45, 2.01 **(b)** 0.28

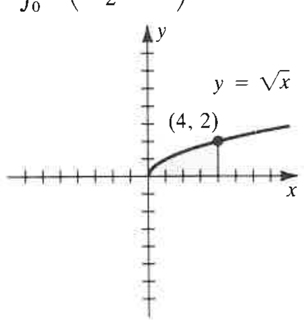


45 $\frac{4}{3}\pi ab^2$ **47 (a)** $p=\frac{r^2}{4h}$ **(b)** $\frac{1}{2}\pi r^2h$

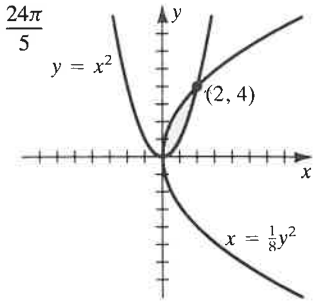
Exercises 5.3

1 $2\pi\int_2^{11} x\sqrt{x-2}\,dx$ **3** $2\pi\int_0^6 y\left(-\frac{1}{2}y+3\right)dy$

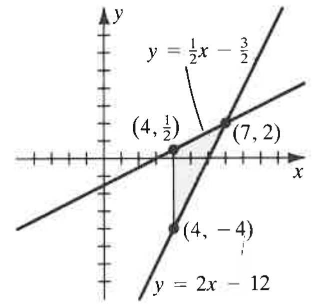
5 $2\pi\int_0^4 x\sqrt{x}\,dx=\frac{128\pi}{5}$



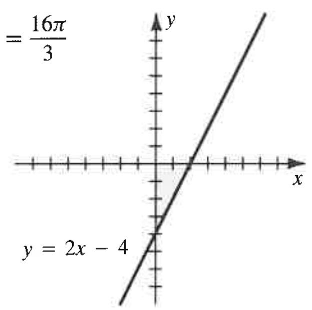
7 $2\pi\int_0^2 x(\sqrt{8x}-x^2)\,dx=\frac{24\pi}{5}$



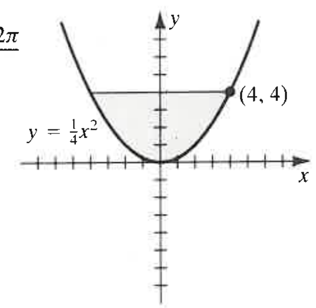
9 $2\pi\int_4^7 x\left[\left(\frac{1}{2}x-\frac{3}{2}\right)-(2x-12)\right]dx=\frac{135\pi}{2}$



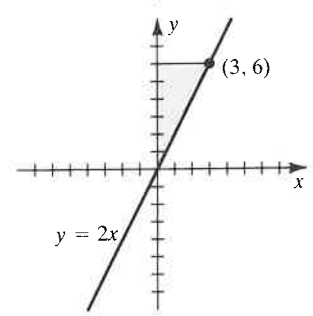
11 $2\pi\int_0^2 x[0-(2x-4)]\,dx=\frac{16\pi}{3}$



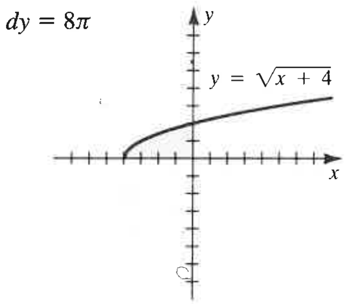
13 $2\cdot2\pi\int_0^4 y\sqrt{4y}\,dy=\frac{512\pi}{5}$



15 $2\pi\int_0^6 y\left(\frac{1}{2}y\right)dy=72\pi$



17 $2\pi\int_0^2 y[0-(y^2-4)]\,dy=8\pi$



19 (a) $2\pi\int_0^2 (3-x)(x^2+1)\,dx$

(b) $2\pi\int_0^2 [x-(-1)](x^2+1)\,dx$

21 (a) $2\cdot2\pi\int_0^4 (4-y)\sqrt{y}\,dy$

(b) $2\cdot2\pi\int_0^4 (5-y)\sqrt{y}\,dy$

(c) $2\pi\int_{-2}^2 (2-x)(4-x^2)\,dx$

(d) $2\pi\int_{-2}^2 [x-(-3)](4-x^2)\,dx$

23 $2\pi\int_0^1 (2-x)[(3-x^2)-(3-x)]\,dx$

25 $2\cdot2\pi\int_{-1}^1 (5-x)\sqrt{1-x^2}\,dx$

27 (a) $2\pi\int_0^{1/2} y(4-1)\,dy+2\pi\int_{1/2}^1 y[(1/y^2)-1]\,dy$

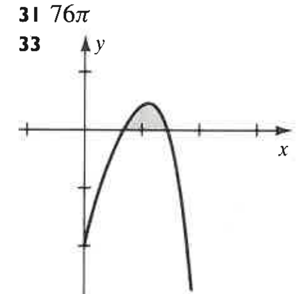
(b) $\pi\int_1^4\left(\frac{1}{\sqrt{x}}\right)^2dx$

29 (a) $2\pi\int_0^1 x(x^2+2)\,dx$

(b) $\pi\int_0^2 (1)^2\,dy+\pi\int_2^3 [(1)^2-(\sqrt{y-2})^2]\,dy$

31 76π

33 **(a)** 0.68, 1.44



(b) $2\pi\int_{0.68}^{1.44} x(-x^4+2.21x^3-3.21x^2+4.42x-2)\,dx$

35 (a) $\frac{8}{3}$ **(b)** 2π **(c)** $\frac{16\pi}{5}$

Exercises 5.4

Exer. 1–26: The first integral represents a general formula for the volume. In Exercises 1–8, the vertical distance between the graphs of $y=\sqrt{x}$ and $y=-\sqrt{x}$ is $[\sqrt{x}-(-\sqrt{x})]$, denoted by $2\sqrt{x}$.

1 $\int_c^d s^2\,dx=\int_0^9 (2\sqrt{x})^2\,dx=162$

3 $\int_c^d \frac{1}{2}\pi r^2\,dx=\int_0^9 \frac{1}{2}\pi(\sqrt{x})^2\,dx=\frac{81\pi}{4}$

5 $\int_c^d \frac{\sqrt{3}}{4}s^2\,dx=\int_0^9 \frac{\sqrt{3}}{4}(2\sqrt{x})^2\,dx=\frac{81\sqrt{3}}{2}$

7 $\int_c^d \frac{1}{2}(B+b)h\,dx$
$=\int_0^9 \frac{1}{2}\left[2\sqrt{x}+\frac{1}{2}(2\sqrt{x})\right]\left[\frac{1}{4}(2\sqrt{x})\right]dx=\frac{243}{8}$

9 $\int_c^d s^2\,dx$
$=2\int_0^a [\sqrt{a^2-x^2}-(-\sqrt{a^2-x^2})]^2\,dx=\frac{16}{3}a^3$

11 $\int_c^d \frac{1}{2}bh\,dx$
$=2\int_0^2 \frac{1}{2}\left[\frac{1}{\sqrt{2}}(4-x^2)\right]\left[\frac{1}{\sqrt{2}}(4-x^2)\right]dx=\frac{128}{15}$

13 $\int_c^d lw\,dx=\int_0^h \left(\frac{2ax}{h}\right)\left(\frac{ax}{h}\right)dx=\frac{2}{3}a^2h$

15 $\int_c^d \frac{1}{2}\pi r^2\,dy=2\int_0^4 \frac{1}{2}\pi\left[\frac{1}{2}\left(4-\frac{1}{4}y^2\right)\right]^2dy=\frac{128\pi}{15}$

17 $\int_c^d lw\,dy=\int_0^a [\sqrt{a^2-y^2}-(-\sqrt{a^2-y^2})]y\,dy$
$=\frac{2}{3}a^3$

19 $\int_c^d \frac{1}{2}bh\,dx=\int_{-a}^a \frac{1}{2}[\sqrt{a^2-x^2}-(-\sqrt{a^2-x^2})]h\,dx$
$=\frac{1}{2}\pi a^2h$

21 $\int_c^d \frac{1}{2}bh\,dx=\int_0^4 \frac{1}{2}\left(\frac{2}{4}x\right)\left(\frac{3}{4}x\right)dx=4\text{ cm}^3$

23 $\int_c^d \frac{1}{2}\pi r^2\,dy=\int_0^a \frac{1}{2}\pi\left[\frac{1}{2}(a-y)\right]^2dy=\frac{\pi}{24}a^3$

25 The areas of cross sections of typical disks and washers are $\pi[f(x)]^2$ and $\pi\{[f(x)]^2-[g(x)]^2\}$, respectively. In each case, the integrand represents $A(x)$ in (5.13).

27 864

Exercises 5.5

1 (a) $\int_1^3 \sqrt{1+(3x^2)^2}\,dx$

(b) $\int_2^{28} \sqrt{1+\left[\frac{1}{3}(y-1)^{-2/3}\right]^2}\,dy$

3 (a) $\int_{-3}^{-1}\sqrt{1+(-2x)^2}\,dx$

(b) $\int_{-5}^{3}\sqrt{1+\left[\frac{1}{2}(4-y)^{-1/2}\right]^2}\,dy$

5 $\int_{1}^{8}\sqrt{1+\left(\frac{4}{9}x^{-1/3}\right)^2}\,dx=\left(4+\frac{16}{81}\right)^{3/2}-\left(1+\frac{16}{81}\right)^{3/2}$ ≈ 7.29

7 $\int_{1}^{4}\sqrt{1+\left(-\frac{3}{2}x^{1/2}\right)^2}\,dx=\frac{8}{27}\left[10^{3/2}-\left(\frac{13}{4}\right)^{3/2}\right]\approx 7.63$

9 $\int_{1}^{2}\sqrt{1+\left(\frac{1}{4}x^2-\frac{1}{x^2}\right)^2}\,dx=\frac{13}{12}$

11 $\int_{1}^{2}\sqrt{1+\left(\frac{3}{2}y^{-4}+\frac{1}{6}y^4\right)^2}\,dy=\frac{353}{240}$

13 $\int_{0}^{2}\sqrt{1+\left(\frac{7}{2}-3y^2\right)^2}\,dy$

15 $8\int_{a}^{1}\sqrt{1+[(-x^{-1/3})(1-x^{2/3})^{1/2}]^2}\,dx=6$, where $a=\left(\frac{1}{2}\right)^{3/2}$

17 (a) $\int_{1}^{1.1}\sqrt{1+\frac{4}{9}x^{-2/3}}\,dx\approx 0.119599$
(b) $\sqrt{13}/30\approx 0.120185$ **(c)** 0.119598

19 (a) $\int_{2}^{2.1}\sqrt{1+4x^2}\,dx\approx 0.422021$
(b) $\sqrt{17}(0.1)\approx 0.412311$ **(c)** $\sqrt{0.1781}\approx 0.422019$

21 (a) $\int_{\pi/6}^{31\pi/180}\sqrt{1+\sin^2 x}\,dx\approx 0.0195733$
(b) $\pi\sqrt{5}/360\approx 0.0195134$ **(c)** 0.0195725

23 9.778303 **25** 1.849432

27 (a) 3.7900; 3.8125; it is smaller
(b) $\int_{0}^{\pi}\sqrt{1+\cos^2 x}\,dx$; 3.8199; 3.8202

29 $2\pi\int_{0}^{1}\sqrt{4x}\sqrt{1+(x^{-1/2})^2}\,dx=\frac{8\pi}{3}(2^{3/2}-1)\approx 15.32$

31 $2\pi\int_{1}^{2}\left(\frac{1}{4}x^4+\frac{1}{8}x^{-2}\right)\sqrt{1+\left(x^3-\frac{1}{4}x^{-3}\right)^2}\,dx$ $=\frac{16{,}911\pi}{1024}\approx 51.88$

33 $2\pi\int_{2}^{4}\frac{1}{8}y^3\sqrt{1+\left(\frac{3}{8}y^2\right)^2}\,dy$ $=\frac{\pi}{27}[8(37)^{3/2}-13^{3/2}]\approx 204.04$

35 $2\pi\int_{4}^{5}\sqrt{25-y^2}\sqrt{1+[(-y)(25-y^2)^{-1/2}]^2}\,dy=10\pi$

37 $2\pi\int_{0}^{h}\left(\frac{r}{h}x\right)\sqrt{1+\left(\frac{r}{h}\right)^2}\,dx=\pi r\sqrt{h^2+r^2}$

39 $2\cdot 2\pi\int_{0}^{r}\sqrt{r^2-x^2}\sqrt{1+[(-x)(r^2-x^2)^{-1/2}]^2}\,dx$ $=4\pi r^2$

41 *Hint*: Regard ds as the slant height of the frustum of a cone that has average radius x.

43 (a) 13.6862; 14.2384; it is smaller
(b) $2\pi\int_{0}^{\pi}\sin x\sqrt{1+\cos^2 x}\,dx$; 13.4821; 14.1937

45 201 in²

47 (a) $x^2=500(y-10)$ **(b)** $\int_{-200}^{200}\sqrt{1+\left(\frac{1}{250}x\right)^2}\,dx$
(c) 282 ft

49 (a) *Hint*: $S=\int_{0}^{a}2\pi x\sqrt{1+\left(\frac{1}{2p}x\right)^2}\,dx$ **(b)** 64,968 ft²

Exercises 5.6

1 (a) and **(b)** 6000 ft-lb **3 (a)** $\frac{128}{3}$ in.-lb **(b)** $\frac{64}{3}$ in.-lb

5 $W_2=3W_1$ **7** 27,945 ft-lb **9** 276 ft-lb **11** 2250 ft-lb

13 (a) $\frac{81\pi}{2}(62.5)\approx 7952$ ft-lb **(b)** $\frac{189\pi}{2}(62.5)\approx 18{,}555$ ft-lb

15 500 ft-lb **17** $575\left(\frac{1}{2}-40^{-1/5}\right)\approx 12.55$ in.-lb

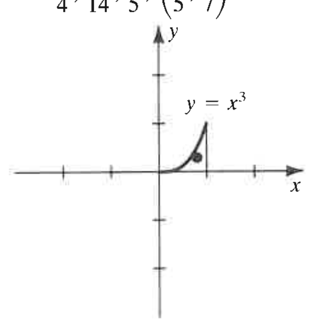
19 $W=\frac{Gm_1m_2h}{(4000)(4000+h)}$ **21** 36.85 ft-lb

23 (a) $\frac{3}{10}k$ J (k a constant) **(b)** $\frac{9}{40}k$ J

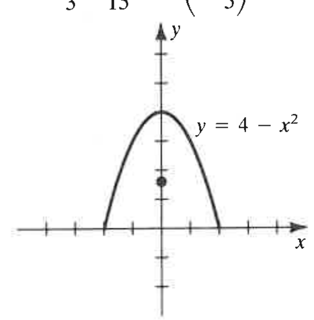
Exercises 5.7

1 250; 140; 0.56 **3** 14; −27; −46; $\left(-\frac{23}{7},-\frac{27}{14}\right)$

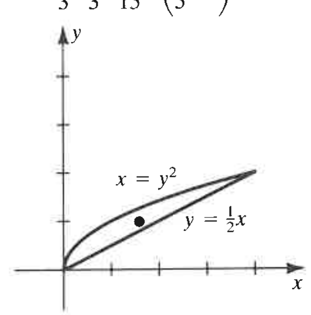
5 $\frac{1}{4}$; $\frac{1}{14}$; $\frac{1}{5}$; $\left(\frac{4}{5},\frac{2}{7}\right)$



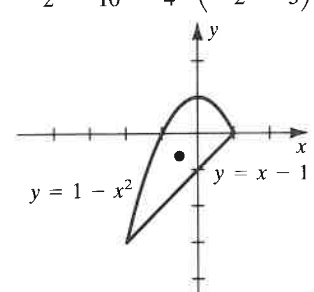
7 $\frac{32}{3}$; $\frac{256}{15}$; 0; $\left(0,\frac{8}{5}\right)$



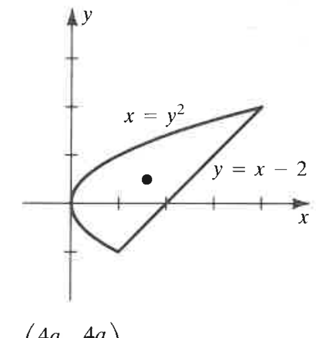
9 $\frac{4}{3}$; $\frac{4}{3}$; $\frac{32}{15}$; $\left(\frac{8}{5},1\right)$



11 $\frac{9}{2}$; $-\frac{27}{10}$; $-\frac{9}{4}$; $\left(-\frac{1}{2},-\frac{3}{5}\right)$



13 $\frac{9}{2}$; $\frac{9}{4}$; $\frac{36}{5}$; $\left(\frac{8}{5},\frac{1}{2}\right)$



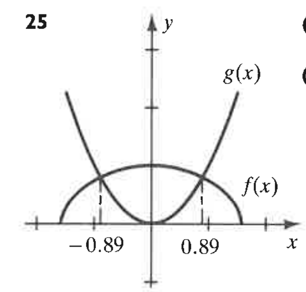
15 $\left(\frac{4a}{3\pi},\frac{4a}{3\pi}\right)$

17 With the center of the circle at the origin, the centroid is $\left(0,-\frac{20a}{3(8+\pi)}\right)$.

19 Show that the centroid is $\left(\frac{1}{3}a,\frac{1}{3}(b+c)\right)$.

21 $(2\pi\cdot 3)(\sqrt{2}\sqrt{18})=36\pi$ **23** $\left(\frac{4a}{3\pi},\frac{4a}{3\pi}\right)$

25



(a) $\rho\int_{-0.89}^{0.89}(\sqrt{|\cos x|}-x^2)\,dx$
(b) 1.19ρ

Exercises 5.8

1 (a) $\frac{1}{2}(62.5)$ lb **(b)** $\frac{3}{2}(62.5)$ lb

3 (a) $\frac{\sqrt{3}}{3}(62.5)$ lb **(b)** $\frac{\sqrt{3}}{24}(62.5)$ lb

5 $\frac{16}{3}(60)$ lb **7** $\frac{592}{3}(62.5)$ lb

9 (a) 90(50) lb **(b)** 54(50) lb; 36(50) lb **11** 1.56 L/min

13 In min: **(a)** 20 **(b)** 66 **(c)** 115 **(d)** 197

15 $10\sqrt{11}-10\approx 23.17$ min

17 666 **19** 11 **21 (a)** and **(b)** 150 J

23 $9-\frac{5\sqrt{5}}{3}\approx 5.27$ gal **25** 1.45 coulombs

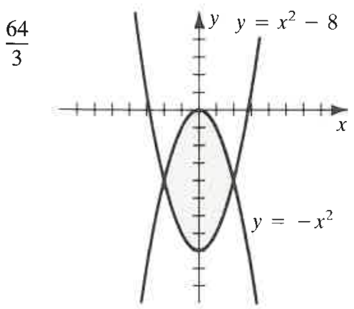
27 (a) $\int_{0}^{1/30}12{,}450\pi\sin(30\pi t)\,dt=830$ cm³
(b) It is not safe, since approximately 0.027 joule is inhaled.

29 32 **31** $x_c=320$; 2560 **33** $x_c=800$; 120,000

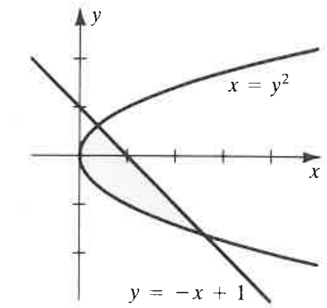
Chapter 5 Review Exercises

1 (a) $2\int_{0}^{2}[(-x^2)-(x^2-8)]\,dx=\frac{64}{3}$

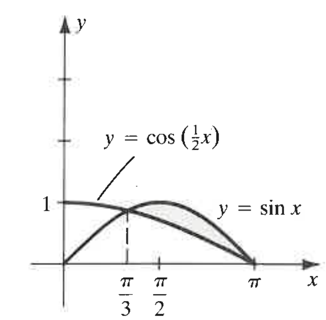
(b) $4\int_{-4}^{0}\sqrt{-y}\,dy=\frac{64}{3}$



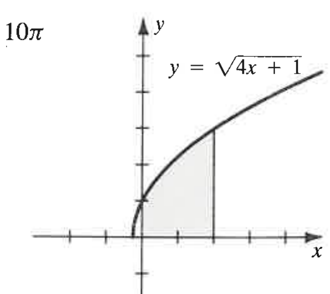
3 $\int_{a}^{b}[(1-y)-y^2]\,dy=\frac{5\sqrt{5}}{6}$, where $a=\frac{1}{2}(-1-\sqrt{5})$ and $b=\frac{1}{2}(-1+\sqrt{5})$



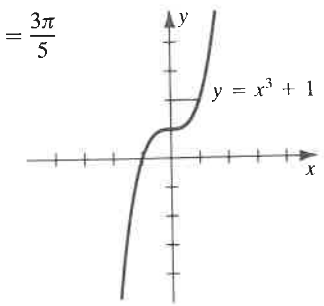
5 $\int_{\pi/3}^{\pi}\left(\sin x-\cos\frac{1}{2}x\right)dx=\frac{1}{2}$



7 $\pi\int_{0}^{2}(\sqrt{4x+1})^2\,dx=10\pi$



9 $2\pi \int_0^1 x[2-(x^3+1)]\,dx = \frac{3\pi}{5}$



11 $2\pi \int_0^{\sqrt{\pi/2}} x(\cos x^2)\,dx = \pi$

13 **(a)** $\pi \int_{-2}^{1} [(-4x+8)^2-(4x^2)^2]\,dx = \frac{1152\pi}{5}$

(b) $2\pi \int_{-2}^{1} (1-x)[(-4x+8)-4x^2]\,dx = 54\pi$

(c) $\pi \int_{-2}^{1} \{(16-4x^2)^2-[16-(-4x+8)]^2\}\,dx$

$= \frac{1728\pi}{5}$

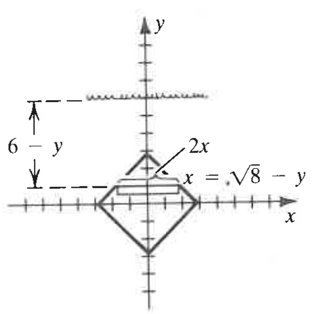
15 $\int_{-2}^{5} \sqrt{1+\left[\frac{1}{3}(x+3)^{-1/3}\right]^2}\,dx$

$= \frac{1}{27}(37^{3/2}-10^{3/2}) \approx 7.16$

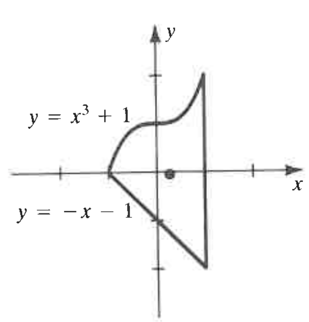
17 $\int_0^4 (5-y)(62.5)\pi(6)^2\,dy = 432\pi(62.5)$ ft-lb

19 $\rho \int_0^{\sqrt{8}} (6-y)2(\sqrt{8}-y)\,dy +$

$\rho \int_{-\sqrt{8}}^{0} (6-y)2(y+\sqrt{8})\,dy = 96(62.5)$ lb



21 $4; -\frac{4}{21}; \frac{16}{15}; \left(\frac{4}{15}, -\frac{1}{21}\right)$



23 $2\pi \int_1^2 \left(\frac{1}{3}x^3+\frac{1}{4}x^{-1}\right)\sqrt{1+\left(x^2-\frac{1}{4}x^{-2}\right)^2}\,dx = \frac{515\pi}{64}$

≈ 25.3

25 900 ft

27 **(a)** The area under the graph of $y = 2\pi x^4$

(b) **(i)** The volume obtained by revolving $y = \sqrt{2}x^2$ about the x-axis

(ii) The volume obtained by revolving $y = x^3$ about the y-axis

(c) The work done by a force of magnitude $y = 2\pi x^4$ as it moves from $x = 0$ to $x = 1$.

29 **(a)** $(a, 0.67)$, $(b, 1.91)$, $a \approx -0.82$, $b \approx 1.38$

(b) $\int_a^b (\sqrt{1+x^3}-x^2)\,dx \approx 1.43$

31 **(a)** $(a, 2.40)$, $(b, 9.53)$, $a \approx 0.29$, $b \approx 4.54$

(b) $\int_a^b [\sqrt{20x}-(x^3-4x^2-x+3)]\,dx \approx 44.42$

CHAPTER • 6

Exercises 6.1

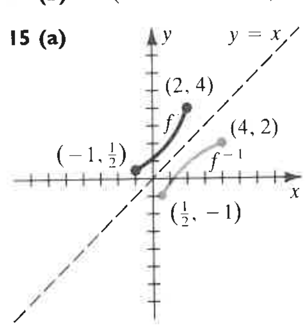
1 $\frac{x-5}{3}$ **3** $\frac{2x+1}{3x}$ **5** $\frac{5x+2}{2x-3}$ **7** $-\frac{1}{3}\sqrt{6-3x}$

9 $3-x^2$, $x \geq 0$ **11** $(x-1)^3$

13 **(a)** The graph of f is a line of slope $a \neq 0$ and hence is one-to-one. $f^{-1}(x) = \frac{x-b}{a}$

(b) No (not one-to-one)

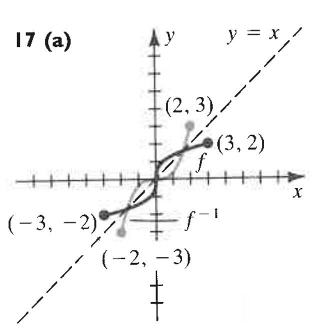
15 **(a)**



(b) $[-1, 2]$; $\left[\frac{1}{2}, 4\right]$

(c) $\left[\frac{1}{2}, 4\right]$; $[-1, 2]$

17 **(a)**



(b) $[-3, 3]$; $[-2, 2]$

(c) $[-2, 2]$; $[-3, 3]$

19 **(a)** $[-0.27, 1.22]$

(b) $[-0.20, 3.31]$; $[-0.27, 1.22]$

21 **(a)** $[-1.43, 1.43]$ **(b)** $[-0.84, 0.84]$; $[-1.43, 1.43]$

23 **(a)** $[-2.14, 1]$ **(b)** $[0.5, 2]$; $[-2.14, 1]$

25 **(a)** f is increasing on $\left[-\frac{3}{2}, \infty\right)$ and hence is one-to-one. **(b)** $[0, \infty)$ **(c)** x

27 **(a)** f is decreasing on $[0, \infty)$ and hence is one-to-one.

(b) $(-\infty, 4]$ **(c)** $-\frac{1}{2\sqrt{4-x}}$

29 **(a)** f is decreasing on $(-\infty, 0)$ and $(0, \infty)$ and hence is one-to-one.

(b) All real numbers except zero **(c)** $-\frac{1}{x^2}$

31 **(a)** f is increasing, since $f'(x) > 0$ for every x **(b)** $\frac{1}{16}$

33 **(a)** f is decreasing, since $f'(x) < 0$ for $x > 0$ **(b)** $-\frac{2}{7}$

35 **(a)** f is increasing, since $f'(x) > 0$ for every x **(b)** $\frac{1}{16}$

Exercises 6.2

1 $\frac{9}{9x+4}$ **3** $\frac{2(3x-1)}{3x^2-2x+1}$ **5** $\frac{2}{2x-3}$ **7** $\frac{15}{3x-2}$

9 $\frac{3x^2}{2x^3-7}$ **11** $1+\ln x$ **13** $\frac{1}{2x}\left(1+\frac{1}{\sqrt{\ln x}}\right)$

15 $-\frac{1}{x}\left[\frac{1}{(\ln x)^2}+1\right]$ **17** $\frac{20}{5x-7}+\frac{6}{2x+3}$

19 $\frac{x}{x^2+1}-\frac{18}{9x-4}$ **21** $\frac{x}{x^2-1}-\frac{x}{x^2+1}$ **23** $\frac{1}{\sqrt{x^2-1}}$

25 $-2\tan 2x$ **27** $9\csc 3x \sec 3x$ **29** $\frac{2\tan 2x}{\ln \sec 2x}$

31 $\tan x$ **33** $\sec x$ **35** $\frac{y(2x^2-1)}{x(3y+1)}$ **37** $\frac{y(y-x\ln y)}{x(x-y\ln x)}$

39 $(5x+2)^2(6x+1)(150x+39)$ **41** $\frac{(14x+11)(x-5)^2}{\sqrt{4x+7}}$

43 $\frac{(19x^2+20x-3)(x^2+3)^4}{2(x+1)^{3/2}}$ **45** $y = 8x-15$

47 $(10, 5\ln 10-5) \approx (10, 6.51)$; $y'' = -(5/x^2) < 0$ implies that the graph is CD for $x > 0$. **49** ± 0.73 yr

51 **(a)** $s'(0) = 0$ m/sec; $s''(0) = \frac{bc}{m_1+m_2}$ m/sec²

(b) $s'\left(\frac{m_2}{b}\right) = c\ln\left(\frac{m_1+m_2}{m_1}\right)$; $s''\left(\frac{m_2}{b}\right) = \frac{bc}{m_1}$

53 The graphs coincide if $x > 0$; however, the graph of $y = \ln(x^2)$ contains points with negative x-coordinates.

55 **(a)** $-3.18 \leq y \leq 0$

(b) x-int.: $\pi/2 \approx 1.57$; max: $f(\pi/2) = 0$

57 **(a)** $1.33 \leq y \leq 2.18$

(b) y-int.: 2

59 **(a)** $-1.97 \leq y \leq 3.79$

(b) x-int.: 0.55; max: $f(2.47) \approx 1.56$, $f(8.14) \approx 2.91$, $f(14.30) \approx 3.49$; min: $f(4.65) \approx 0.34$, $f(10.97) \approx 1.19$, $f(17.26) \approx 1.65$

61 0.5671 **63** −3.2088, 2.0435 **65** 1.7477 **67** 1.8929

69 12.0536 **71** 9.3392

Exercises 6.3

1 $-5e^{-5x}$ **3** $6xe^{3x^2}$ **5** $\frac{e^{2x}}{\sqrt{1+e^{2x}}}$ **7** $\frac{e^{\sqrt{x+1}}}{2\sqrt{x+1}}$

9 $2xe^{-2x}(1-x)$ **11** $\frac{e^x(x-1)^2}{(x^2+1)^2}$ **13** $12e^{4x}(e^{4x}-5)^2$

15 $-\frac{e^{1/x}}{x^2}-e^{-x}$ **17** $\frac{4}{(e^x+e^{-x})^2}$ **19** $e^{-2x}\left(\frac{1}{x}-2\ln x\right)$

21 $5e^{5x}\cos e^{5x}$ **23** $e^{-x}\tan e^{-x}$

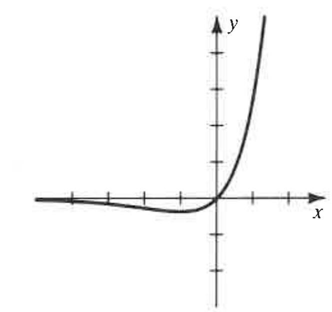
25 $e^{3x}\left(\frac{\sec^2\sqrt{x}}{2\sqrt{x}}+3\tan\sqrt{x}\right)$

27 $-8e^{-4x}\sec^2(e^{-4x})\tan(e^{-4x})$ **29** $e^{\cot x}(1-x\csc^2 x)$

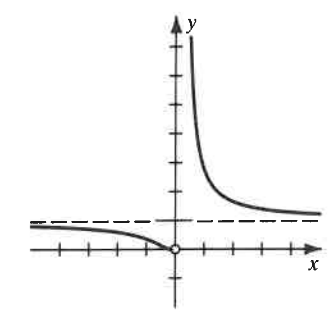
31 $\frac{3x^2-ye^{xy}}{xe^{xy}+6y}$ **33** $\frac{e^x\cot y-e^{2y}}{2xe^{2y}+e^x\csc^2 y}$

35 $y = (e+3)x-(e+1)$

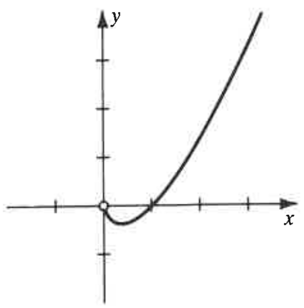
37 Min: $f(-1) = -e^{-1} \approx -0.368$; increasing on $[-1, \infty)$; decreasing on $(-\infty, -1]$; CU on $(-2, \infty)$; CD on $(-\infty, -2)$; PI: $(-2, -2e^{-2}) \approx (-2, -0.271)$



39 Decreasing on $(-\infty, 0)$ and $(0, \infty)$; CU on $\left(-\frac{1}{2}, 0\right)$ and $(0, \infty)$; CD on $\left(-\infty, -\frac{1}{2}\right)$; PI: $\left(-\frac{1}{2}, e^{-2}\right)$



41 Min: $f(e^{-1}) = -e^{-1}$; increasing on $[e^{-1}, \infty)$; decreasing on $(0, e^{-1}]$; CU on $(0, \infty)$; no PI

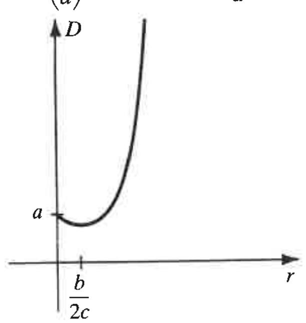


43 $q'(t) = -cq(t)$ **45 (a)** $\frac{\ln(a/b)}{a-b}$ **(b)** $\lim_{t\to\infty} C(t) = 0$

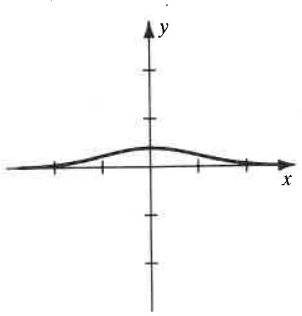
47 (a) 75.8 cm; 15.98 cm/yr **(b)** 3 mo; 6 yr

49 (a) $f\left(\frac{n}{a}\right)$ **(b)** At $x = \frac{2}{a}$

51



55 Max: $f(\mu) = \frac{1}{\sigma\sqrt{2\pi}}$; increasing on $(-\infty, \mu]$; decreasing on $[\mu, \infty)$; CU on $(-\infty, \mu - \sigma)$ and $(\mu + \sigma, \infty)$; CD on $(\mu - \sigma, \mu + \sigma)$; PI: $\left(\mu \pm \sigma, \frac{1}{\sigma\sqrt{2\pi e}}\right)$; both limits equal 0



57 (a) $[0.054, 1]$ **(b)** y-int.: 1

59 (a) $[-3.18, 6.13]$
(b) x-int.: ± 0.84, 2.52, 4.20, 5.88, 7.56; y-int.: 6; max: $f(-0.11) \approx 6.13$, $f(3.25) \approx 1.65$, $f(6.61) \approx 0.45$; min: $f(1.57) \approx -3.18$, $f(4.93) \approx -0.86$

61 0.5671 **63** 1.2022 **65** $e^{-1/2} \approx 0.607$

Exercises 6.4

1 (a) $\frac{1}{2}\ln|2x+7| + C$ **(b)** $\ln\sqrt{3}$

3 (a) $2\ln|x^2 - 9| + C$ **(b)** $\ln\frac{25}{64}$

5 (a) $-\frac{1}{4}e^{-4x} + C$ **(b)** $-\frac{1}{4}(e^{-12} - e^{-4})$

7 (a) $-\frac{1}{2}\ln|\cos 2x| + C$ **(b)** $\frac{1}{4}\ln 2$

9 (a) $2\ln\left|\csc\frac{1}{2}x - \cot\frac{1}{2}x\right| + C$ **(b)** $2\ln(2+\sqrt{3})$

11 $\frac{1}{2}\ln|x^2 - 4x + 9| + C$

13 $\frac{1}{2}x^2 + 4x + 4\ln|x| + C$

15 $\frac{1}{2}(\ln x)^2 + C$ **17** $\frac{1}{2}x^2 + \frac{1}{5}e^{5x} + C$

19 $-\frac{3}{2}\ln|1 + 2\cos x| + C$ **21** $e^x + 2x - e^{-x} + C$

23 $\ln(e^x + e^{-x}) + C$ **25** $3\ln|\sin\sqrt[3]{x}| + C$

27 $\frac{1}{2}\ln|\sec 2x + \tan 2x| + C$ **29** $-\frac{1}{3}\ln|\sec e^{-3x}| + C$

31 $\ln|\csc x - \cot x| + \cos x + C$ **33** $\ln|\csc x| + C$

35 $x + 2\ln|\sec x + \tan x| + \tan x + C$ **37** 4

39 $\pi(1 - e^{-1})$ **41** $y = 2e^{2x} - \frac{3}{2}e^{-2x} + \frac{7}{2}$

43 $y = 3e^{-x} + 4x - 4$ **45** $\frac{2}{\ln(13/4)} \approx 1.697$

47 (a) 25 **(b)** 205 **(c)** 12 **49** $\Delta S = c\ln\frac{T_2}{T_1}$

51 (a) $\frac{5}{2}(1 - e^{-4t})$ **(b)** $\lim_{t\to\infty} Q(t) = \frac{5}{2}$ coulombs

53 (a) $s(t) = kv_0(1 - e^{-t/k})$ **(b)** $\lim_{t\to\infty} s(t) = kv_0$

55 0.7468 **57** 127.2930 **59** 6.43 **61** 9.34

Exercises 6.5

1 $7^x \ln 7$ **3** $8^{x^2+1}(2x\ln 8)$ **5** $\frac{4x^3 + 6x}{(x^4 + 3x^2 + 1)\ln 10}$

7 $5^{3x-4}(3\ln 5)$ **9** $\frac{-(x^2+1)10^{1/x}(\ln 10)}{x^2} + (2x)10^{1/x}$

11 $\frac{30x}{(3x^2+2)\ln 10}$ **13** $\left(\frac{6}{6x+4} - \frac{2}{2x-3}\right)\frac{1}{\ln 5}$

15 $\frac{1}{x\ln x\ln 10}$ **17** $ex^{e-1} + e^x$

19 $(x+1)^x\left[\frac{x}{x+1} + \ln(x+1)\right]$ **21** $2^{\sin^2 x}(\sin 2x)\ln 2$

23 (a) 0 **(b)** $5x^4$ **(c)** $\sqrt{5}x^{\sqrt{5}-1}$ **(d)** $(\sqrt{5})^x \ln\sqrt{5}$
(e) $x^{1+x^2}(1 + 2\ln x)$

29 (a) $\frac{7^x}{\ln 7} + C$ **(b)** $\frac{342}{49\ln 7} \approx 3.59$

31 (a) $\frac{-5^{-2x}}{2\ln 5} + C$ **(b)** $\frac{12}{625\ln 5} \approx 0.012$ **33** $\frac{10^{3x}}{3\ln 10} + C$

35 $\frac{-3^{-x^2}}{2\ln 3} + C$ **37** $\frac{\ln(2^x+1)}{\ln 2} + C$

39 $(\ln 10)\ln|\log x| + C$ **41** $-\frac{3^{\cos x}}{\ln 3} + C$

43 (a) $\pi^\pi x + C$ **(b)** $\frac{1}{5}x^5 + C$ **(c)** $\frac{x^{\pi+1}}{\pi+1} + C$

(d) $\frac{\pi^x}{\ln \pi} + C$ **45** $\frac{1}{\ln 2} - \frac{1}{2} \approx 0.94$

47 (a) \$0.05/yr **(b)** \$0.95

49 (a) ln trout/yr: 95; 62; 53 **(b)** 9.36

51 pH ≈ 2.201; $\pm 0.1\%$

53 (b) $S = \frac{k}{x}$, where $k = \frac{a}{\ln 10}$;
$S(x) = 2\,S(2x)$ (twice as sensitive)

55 (a) With $n = r/h$,

$$\ln A = \ln[P(1+h)^{rt/h}] = \ln P + rt\ln(1+h)^{1/h}.$$

(b) Since $h = r/n$, $n \to \infty$ if and only if $h \to 0^+$. Thus,

$$\ln A = \lim_{h\to 0^+}[\ln P + rt\ln(1+h)^{1/h}] = \ln P + rt\ln e = \ln(Pe^{rt})$$

and $A = Pe^{rt}$.

57 Let $h = x/n$. Then

$$\lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n = \lim_{h\to 0^+}(1+h)^{x/h} = \lim_{h\to 0^+}[(1+h)^{1/h}]^x = e^x.$$

Exercises 6.6

1 $q(t) = 5000(3)^{t/10}$; 45,000; $\frac{10\ln 10}{\ln 3} \approx 20.96$ hr

3 $30\left(\frac{29}{30}\right)^5 \approx 25.32$ in.

5 $\frac{\ln(40/5.5)}{0.02} \approx 99.21$ yr after Jan. 1, 1993 (March 17, 2092)

7 $\frac{5\ln(1/6)}{\ln(1/3)} \approx 8.15$ min

9 Proceed as in the solution to Example 1.

11 $P(z) = \left(\frac{288 - 0.01z}{288}\right)^{3.42}$ **13** $\frac{29\ln(2/5)}{\ln(1/2)} \approx 38.34$ yr

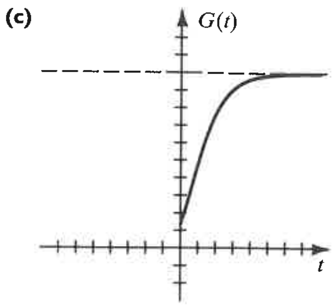
15 $600\left(\frac{1}{2}\right)^{-3/16} \approx 683.27$ mg

17 $v(y) = \sqrt{2k\left(\frac{1}{y} - \frac{1}{y_0}\right) + v_0^2}$

19 $\frac{5700\ln(0.2)}{\ln(1/2)} \approx 13{,}235$ yr **21** Use Theorem (4.35).

23 $V(t) = \frac{1}{27}(kt + C)^3$

25 (c)



Exercises 6.7

1 (a) $-\frac{\pi}{4}$ **(b)** $\frac{2\pi}{3}$ **(c)** $-\frac{\pi}{3}$

3 (a) Not defined **(b)** Not defined **(c)** $\frac{\pi}{4}$

5 (a) $\frac{\pi}{3}$ **(b)** $\frac{5\pi}{6}$ **(c)** $-\frac{\pi}{6}$ **7 (a)** $\frac{\pi}{3}$ **(b)** $\frac{2\pi}{3}$ **(c)** $\frac{\pi}{6}$

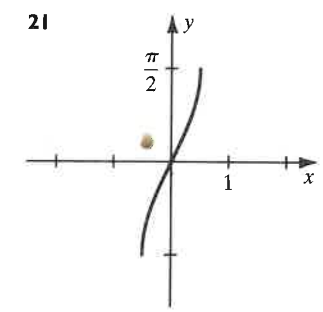
9 (a) $\frac{\sqrt{3}}{2}$ **(b)** 0 **(c)** Not defined

11 (a) $-\frac{\sqrt{21}}{2}$ **(b)** $\frac{\sqrt{65}}{4}$ **(c)** $\frac{5}{\sqrt{24}}$

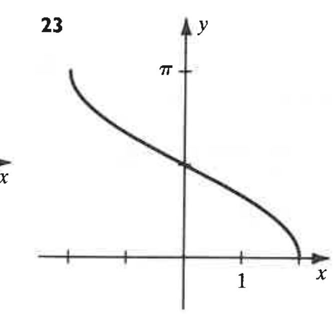
13 (a) -1.1971 **(b)** 0.2712 **15 (a)** 1.0556 **(b)** 0.6183

17 $\frac{x}{\sqrt{x^2+1}}$ **19** $\frac{3}{\sqrt{9-x^2}}$

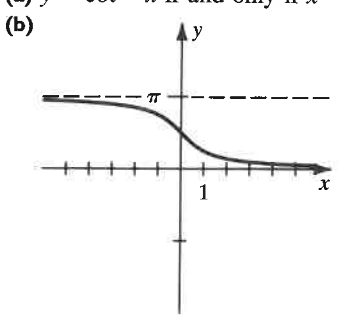
21



23



27 (a) $y = \cot^{-1} x$ if and only if $x = \cot y$ for $0 < y < \pi$.
(b)



29 (a) $\alpha = \theta - \sin^{-1}\frac{d}{k}$ **(b)** 40° **31** $\frac{1}{2\sqrt{x}\sqrt{1-x}}$

33 $\dfrac{3}{9x^2 - 30x + 26}$ **35** $\dfrac{-e^{-x}}{\sqrt{e^{-2x} - 1}} - e^{-x} \operatorname{arcsec} e^{-x}$

37 $\dfrac{2x}{(1 + x^4) \arctan (x^2)}$ **39** $-\dfrac{9(1 + \cos^{-1} 3x)^2}{\sqrt{1 - 9x^2}}$

41 $\left(\dfrac{1}{x^2}\right) \sin \left(\dfrac{1}{x}\right) + \sec x \tan x - \dfrac{1}{\sqrt{1 - x^2}}$

43 $3^{\arcsin (x^3)} \dfrac{(3 \ln 3)x^2}{\sqrt{1 - x^6}}$

45 $\dfrac{1 - 2x \arctan x}{(x^2 + 1)^2}$ **47** $\dfrac{1}{2\sqrt{x}}\left(\dfrac{1}{\sqrt{x - 1}} + \sec^{-1}\sqrt{x}\right)$

49 $\dfrac{ye^x - 2x - \sin^{-1} y}{\dfrac{x}{\sqrt{1 - y^2}} - e^x}$

51 (a) $\dfrac{1}{4} \tan^{-1}\left(\dfrac{x}{4}\right) + C$ **(b)** $\dfrac{\pi}{16}$

53 (a) $\dfrac{1}{2}\sin^{-1} (x^2) + C$ **(b)** $\dfrac{\pi}{12}$

55 $2 \tan^{-1}\sqrt{x} + C$ **57** $\sin^{-1} \left(\dfrac{e^x}{4}\right) + C$

59 $\dfrac{1}{2} \ln (x^2 + 9) + C$ **61** $\dfrac{1}{5} \sec^{-1}\left(\dfrac{e^x}{5}\right) + C$

63 $\pm\dfrac{7}{3576}$ rad **65** $-\dfrac{25}{1044}$ rad/sec **67** $\sqrt{4800} \approx 69.3$ ft

69 $\dfrac{2\pi}{27} \approx 0.233$ mi/sec **75** $x \sin^{-1} (2x) + \dfrac{1}{2} \sqrt{1 - 4x^2} + C$

77 $\dfrac{1}{2}x^2 \tan^{-1} (x^2) - \dfrac{1}{4} \ln (x^4 + 1) + C$

79 0.7241 **81** 2.0570 **83** 31.9285

Exercises 6.8

1 (a) 27.2899 **(b)** 2.1250 **(c)** −0.9951
(d) 1.0000 **(e)** 0.2658 **(f)** −0.8509

3 $5 \cosh 5x$ **5** $3x^2 \sinh (x^3)$

7 $\dfrac{1}{2\sqrt{x}}(\sqrt{x} \operatorname{sech}^2 \sqrt{x} + \tanh \sqrt{x})$

9 $\left(\dfrac{1}{x^2}\right) \operatorname{csch}^2 \left(\dfrac{1}{x}\right)$

11 $\dfrac{-2x \operatorname{sech} (x^2)[(x^2 + 1) \tanh (x^2) + 1]}{(x^2 + 1)^2}$

13 $-12 \operatorname{csch}^2 6x \coth 6x$

15 (a) $\mathbb{R}$ **(b)** $\dfrac{4x \sinh \sqrt{4x^2 + 3}}{\sqrt{4x^2 + 3}}$

17 (a) $\mathbb{R}$ **(b)** $-\dfrac{\operatorname{sech}^2 x}{(\tanh x + 1)^2}$

19 $\dfrac{1}{3} \sinh (x^3) + C$ **21** $2 \cosh \sqrt{x} + C$

23 $\dfrac{1}{3} \tanh 3x + C$ **25** $-2 \coth \left(\dfrac{1}{2}x\right) + C$

27 $-\dfrac{1}{3} \operatorname{sech} 3x + C$ **29** $-\operatorname{csch} x + C$

31 $(\ln (2 \pm \sqrt{3}), \pm\sqrt{3})$

33 Show that $A = \dfrac{1}{2}(\cosh t)(\sinh t) - \displaystyle\int_1^{\cosh t} \sqrt{x^2 - 1}\, dx$

and that $\dfrac{dA}{dt} = \dfrac{1}{2}$.

35 (a) 286,574 ft² **(b)** 1494 ft **37** 34.94 ft

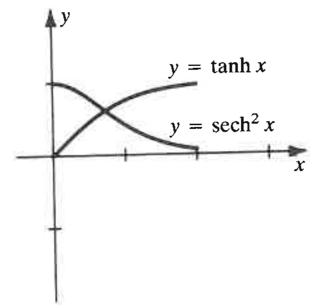
39 $10.5 \sinh^{-1} \frac{4}{3} \approx 11.54$ ft

41 (b) $y = \dfrac{1}{\alpha} \ln [\cosh (\sqrt{g\alpha}\, t + v_0)] + h_0$

43 (a) $\displaystyle\lim_{h\to\infty} v^2 = \dfrac{gL}{2\pi}$ **(b)** *Hint*: Let $f(h) = v^2$.

(a) 0.7 (b) 0.722

45



47 $\cosh x + \sinh x = \dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2} = e^x$

49 $\sinh x \cosh y + \cosh x \sinh y$

$$= \frac{(e^x - e^{-x})(e^y + e^{-y})}{4} + \frac{(e^x + e^{-x})(e^y - e^{-y})}{4}$$

$$= \frac{(e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}) + (e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y})}{4}$$

$$= \frac{2e^{x+y} - 2e^{-x-y}}{4} = \frac{e^{x+y} - e^{-(x+y)}}{2} = \sinh (x + y)$$

51 $\sinh(x - y)$
$= \sinh(x + (-y))$
$= \sinh x \cosh(-y) + \cosh x \sinh(-y)$ (Exer. 49)
$= \sinh x \cosh y - \cosh x \sinh y$ (Exer. 48)

53 Let $y = x$ in Exercise 49.

55 From Exercise 54,

$$\cosh 2y = \cosh^2 y + \sinh^2 y = (1 + \sinh^2 y) + \sinh^2 y = 1 + 2 \sinh^2 y,$$

and hence

$$\sinh^2 y = \frac{\cosh 2y - 1}{2}.$$

Let $y = \dfrac{x}{2}$ to obtain the identity.

57 $\cosh nx + \sinh nx = \dfrac{e^{nx} + e^{-nx}}{2} + \dfrac{e^{nx} - e^{-nx}}{2}$

$= e^{nx} = (e^x)^n = (\cosh x + \sinh x)^n$

59 (a) 0.8814 **(b)** 1.3170 **(c)** −0.5493 **(d)** 1.3170

61 $\dfrac{5}{\sqrt{25x^2 + 1}}$ **63** $\dfrac{1}{2\sqrt{x}\sqrt{x - 1}}$ **65** $\dfrac{4}{16x^2 - 1}$

67 $-\dfrac{2}{x\sqrt{1 - x^4}}$

69 (a) $(\frac{1}{4}, \infty)$ **(b)** $\dfrac{4}{\sqrt{16x^2 - 1} \cosh^{-1} (4x)}$

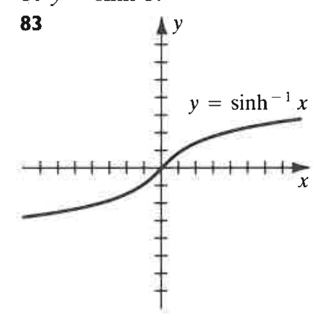
71 (a) $(-2, 0)$ **(b)** $-\dfrac{1}{x(x + 2)}$

73 $\dfrac{1}{4} \sinh^{-1} \left(\dfrac{4}{9}x\right) + C$ **75** $\dfrac{1}{14} \tanh^{-1} \left(\dfrac{2}{7}x\right) + C$

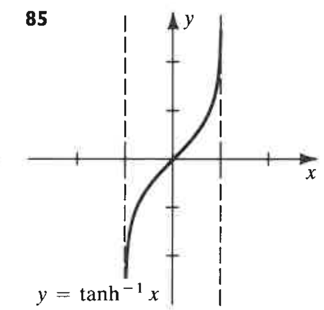
77 $\cosh^{-1} \left(\dfrac{e^x}{4}\right) + C$ **79** $-\dfrac{1}{6} \operatorname{sech}^{-1} \left(\dfrac{x^2}{3}\right) + C$

81 $y = \sinh 3t$

83



85



Exer. 87–91: (a) Use a procedure similar to that given in the text for $\sinh^{-1} x$. **(b)** Let $u = x$ in Theorem (6.48) and differentiate $\cosh^{-1} u$. **(c)** Differentiate the right-hand side.

Exercises 6.9

1 $\dfrac{1}{2}$ **3** $\dfrac{1}{40}$ **5** $\dfrac{3}{13}$ **7** $-\dfrac{1}{2}$ **9** $\dfrac{1}{6}$ **11** ∞ **13** $\dfrac{1}{3}$ **15** 1

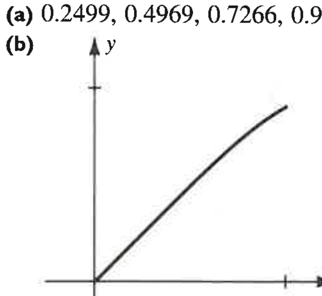
17 ∞ **19** $\dfrac{2}{5}$ **21** 0 **23** ∞ **25** 2 **27** $\dfrac{3}{5}$ **29** −3

31 0 **33** ∞ **35** 1

37 0.9129, 0.9901, 0.9990, 0.9999; predict limit of 1

39 gt **41** $\dfrac{1}{2}A\omega_0 t \sin \omega_0 t$ **43 (a)** 1 **(b)** $-\dfrac{1}{18}$

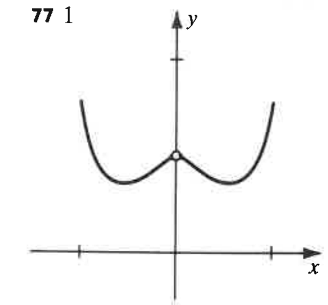
45 (a) 0.2499, 0.4969, 0.7266, 0.9045
(b)



49 0 **51** 0 **53** 0 **55** 1 **57** e^5 **59** 1 **61** ∞

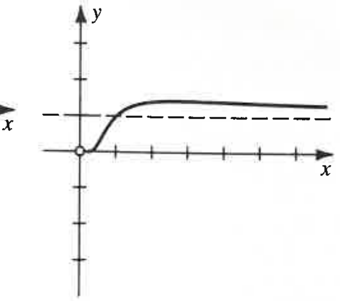
63 e^2 **65** 0 **67** $-\infty$ **69** $\dfrac{1}{2}$ **71** e **73** $e^{1/3}$ **75** ∞

77 1



79 (a) Max: $f(e) = e^{1/e} \approx 1.44$; $\displaystyle\lim_{x\to 0^+} x^{1/x} = 0$

(b) $y = 1$



83 gt

Chapter 6 Review Exercises

1 $\dfrac{10 - x}{15}$

3 f is decreasing, since $f'(x) < 0$ for $-1 \le x \le 1$; $-\dfrac{1}{8}$

5 $\dfrac{75x^2}{5x^3 - 4}$ **7** $\dfrac{12}{3x + 2} + \dfrac{3}{6x - 5} - \dfrac{8}{8x - 7}$

9 $\dfrac{-4x}{(2x^2 + 3)[\ln(2x^2 + 3)]^2}$ **11** $2x$

13 $\dfrac{10^x}{x \ln 10} + 10^x(\ln 10) \log x$ **15** $\dfrac{1}{4x\sqrt{\ln\sqrt{x}}}$

17 $2xe^{-x^2}(1 - x^2)$ **19** $\dfrac{10^{\ln x} \ln 10}{x}$ **21** $\dfrac{2 \ln x\, (x^{\ln x})}{x}$

23 $2e^{-2x} \csc e^{-2x}(\csc^2 e^{-2x} + \cot^2 e^{-2x})$

25 $-16 \tan 4x$ **27** $-\dfrac{y}{x}$

29 $\left[\dfrac{4}{3(x + 2)} + \dfrac{3}{2(x - 3)}\right](x + 2)^{4/3}(x - 3)^{3/2}$

31 (a) $-2e^{-\sqrt{x}} + C$ **(b)** $2(e^{-1} - e^{-2}) \approx 0.465$

33 $-\dfrac{1}{2} \ln |\cos x^2| + C$ **35** $\dfrac{x^{e+1}}{e + 1} + C$

37 $-\dfrac{1}{2}e^{-2x} - 2e^{-x} + x + C$

39 $\dfrac{1}{2}x^2 - 2x + 4 \ln |x + 2| + C$ **41** $-\dfrac{1}{8}e^{4/x^2} + C$

43 $\ln(1 + e^x) + C$ **45** $\dfrac{(5e)^x}{\ln(5e)} + C$

47 $\cos e^{-x} + C$ **49** $-\ln |1 + \cot x| + C$

51 $-\ln | \cos e^x | + C$

53 $-\dfrac{1}{3} \cot 3x + \dfrac{2}{3} \ln |\csc 3x - \cot 3x| + x + C$

55 $y = -\dfrac{1}{9}e^{-3x} + \dfrac{5}{3}x - \dfrac{8}{9}$ **57** $4e^2 + 12 \approx 41.56$ cm

59 $y - e = -2(1 + e)(x - 1)$

61 $\dfrac{\pi}{8}(e^{-16} - e^{-24}) \approx 4.42 \times 10^{-8}$

63 $\frac{5\ln(1/100)}{\ln(1/2)} \approx 33.2$ days

65 (a) $\frac{3\ln(3/10)}{\ln(1/2)} \approx 5.2$ hr or 2.2 additional hr

(b) $10\left[1-\left(\frac{1}{2}\right)^{7/3}\right] \approx 8.016$ lb

67 $100{,}000(2)^6 = 6{,}400{,}000$

69 $\frac{1}{2x\sqrt{x-1}}$ **71** $\frac{2x}{\sqrt{x^4-1}} + 2x\,\text{arcsec}\,(x^2)$

73 $\frac{2x}{(1+x^4)\tan^{-1}(x^2)}$ **75** $\frac{-x}{\sqrt{x^2(1-x^2)}}$

77 $\frac{1}{(1+x^2)[1+(\tan^{-1}x)^2]}$ **79** $-5e^{-5x}\sinh e^{-5x}$

81 $(\cosh x - \sinh x)^{-2}$, or e^{2x} **83** $\frac{2x}{\sqrt{x^4+1}}$

85 $\frac{1}{6}\tan^{-1}\left(\frac{3}{2}x\right)+C$ **87** $-\sqrt{1-e^{2x}}+C$

89 $\frac{1}{2}\sinh(x^2)+C$ **91** $\frac{\pi}{3}$

93 $\frac{1}{2}\sin^{-1}\left(\frac{2}{3}x\right)+C$ **95** $-\frac{1}{3}\,\text{sech}^{-1}\left(\frac{2}{3}|x|\right)+C$

97 $\frac{1}{25}\sqrt{25x^2+36}+C$ **99** $\left(\pm\frac{4}{15}, \sin^{-1}\left(\pm\frac{4}{5}\right)\right)$

101 Let $c=\tan^{-1}\frac{1}{2}$. Min: $f(c)=5\sqrt{5}$; increasing on $\left[c,\frac{\pi}{2}\right)$; decreasing on $(0,c]$.

103 (a) $\frac{1}{2}\tan^{-1}4+\frac{\pi}{2}n$ for $n=0, 1, 2, 3$

(b) 0.66, 2.23, 3.80, 5.38

105 $\frac{1}{260}$ rad/sec $\approx 0.22°$/sec

107 $-\frac{800}{2581}\approx -0.31$ rad/sec **109** $\frac{1}{2}\ln 2$ **111** ∞

113 0 **115** $-\infty$ **117** e **119** 1 **121** 0

CHAPTER 7

Exercises 7.1

1 $-(x+1)e^{-x}+C$ **3** $\frac{1}{27}e^{3x}(9x^2-6x+2)+C$

5 $\frac{1}{5}x\sin 5x+\frac{1}{25}\cos 5x+C$

7 $x\sec x - \ln|\sec x+\tan x|+C$

9 $x^2\sin x+2x\cos x-2\sin x+C$

11 $x\tan^{-1}x-\frac{1}{2}\ln(1+x^2)+C$

13 $\frac{2}{9}x^{3/2}(3\ln x-2)+C$ **15** $-x\cot x+\ln|\sin x|+C$

17 $-\frac{1}{2}e^{-x}(\sin x+\cos x)+C$

19 $\cos x(1-\ln\cos x)+C$

21 $-\frac{1}{2}\csc x\cot x+\frac{1}{2}\ln|\csc x-\cot x|+C$

23 $\frac{1}{3}(2-\sqrt{2})\approx 0.20$ **25** $\frac{\pi}{4}$

27 $\frac{1}{40{,}400}(2x+3)^{100}(200x-3)+C$

29 $\frac{1}{41}e^{4x}(4\sin 5x-5\cos 5x)+C$

31 $x(\ln x)^2-2x\ln x+2x+C$

33 $x^3\cosh x-3x^2\sinh x+6x\cosh x-6\sinh x+C$

35 $2\sqrt{x}\sin\sqrt{x}+2\cos\sqrt{x}+C$

37 $x\cos^{-1}x-\sqrt{1-x^2}+C$ **39** Let $u=x^m$.

41 Let $u=(\ln x)^m$.

43 $e^x(x^5-5x^4+20x^3-60x^2+120x-120)+C$

45 2π **47** $\frac{\pi}{2}(e^2+1)\approx 13.18$ **49** $\left(\frac{3\ln 3-2}{2}, 1\right)$

Exercises 7.2

1 $\sin x-\frac{1}{3}\sin^3 x+C$ **3** $\frac{3}{8}x-\frac{1}{32}\sin 4x+C$

5 $-\frac{1}{3}\cos^3 x+\frac{1}{5}\cos^5 x+C$

7 $\frac{1}{8}\left(\frac{5}{2}x-2\sin 2x+\frac{3}{8}\sin 4x+\frac{1}{6}\sin^3 2x\right)+C$

9 $\frac{1}{4}\tan^4 x+\frac{1}{6}\tan^6 x+C$ **11** $\frac{1}{5}\sec^5 x-\frac{1}{3}\sec^3 x+C$

13 $\frac{1}{5}\tan^5 x-\frac{1}{3}\tan^3 x+\tan x-x+C$

15 $\frac{2}{3}\sin^{3/2}x-\frac{2}{7}\sin^{7/2}x+C$ **17** $\tan x-\cot x+C$

19 $\frac{2}{3}-\frac{5}{6\sqrt{2}}\approx 0.08$ **21** $\frac{1}{2}\left(\frac{1}{2}\sin 2x-\frac{1}{8}\sin 8x\right)+C$

23 $\frac{3}{5}$ **25** $-\frac{1}{5}\cot^5 x-\frac{1}{7}\cot^7 x+C$

27 $-\ln(2-\sin x)+C$ **29** $-\frac{1}{1+\tan x}+C$

31 $\frac{3}{4}\pi^2\approx 7.40$ **33** $\frac{5}{2}$

35 (a) Use the trigonometric product-to-sum formulas.

(b)
$$\int \sin mx\cos nx\,dx = \begin{cases} -\frac{\cos(m+n)x}{2(m+n)}-\frac{\cos(m-n)x}{2(m-n)}+C & \text{if } m\ne n\\ -\frac{\cos 2mx}{4m}+C & \text{if } m=n\end{cases}$$
$$\int\cos mx\cos nx\,dx = \begin{cases}\frac{\sin(m+n)x}{2(m+n)}+\frac{\sin(m-n)x}{2(m-n)}+C & \text{if } m\ne n\\ \frac{x}{2}+\frac{\sin 2mx}{4m}+C & \text{if } m=n\end{cases}$$

Exercises 7.3

1 $\frac{1}{2}\ln\left|\frac{2}{x}-\frac{\sqrt{4-x^2}}{x}\right|+C$ **3** $\frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}}{x}-\frac{3}{x}\right|+C$

5 $\frac{\sqrt{x^2-25}}{25x}+C$ **7** $-\sqrt{4-x^2}+C$

9 $-\frac{x}{\sqrt{x^2-1}}+C$ **11** $\frac{1}{432}\left[\tan^{-1}\left(\frac{x}{6}\right)+\frac{6x}{x^2+36}\right]+C$

13 $\sin^{-1}\left(\frac{x}{3}\right)+C$ **15** $\frac{1}{2(16-x^2)}+C$

17 $\frac{1}{243}(9x^2+49)^{3/2}-\frac{49}{81}\sqrt{9x^2+49}+C$

19 $\frac{(3+2x^2)\sqrt{x^2-3}}{27x^3}+C$ **21** $-\frac{8}{x^2}+8\ln|x|+\frac{1}{2}x^2+C$

23 $25\pi[\sqrt{2}-\ln(\sqrt{2}+1)]\approx 41.85$ **25** 509×10^6 km²

27 $y=\sqrt{x^2-16}-4\sec^{-1}\frac{x}{4}$

29 Let $u=a\tan\theta$. **31** Let $u=a\sin\theta$.

33 Let $u=a\sec\theta$.

Exercises 7.4

Answers are expressed as sums that correspond to partial fraction decompositions. Logarithms can be combined. Thus, an equivalent answer for Exercise 1 is $\ln|x|^3(x-4)^2+C$.

1 $3\ln|x|+2\ln|x-4|+C$

3 $4\ln|x+1|-5\ln|x-2|+\ln|x-3|+C$

5 $6\ln|x-1|+\frac{5}{x-1}+C$

7 $3\ln|x-2|-2\ln|x+4|+C$

9 $2\ln|x|-\ln|x-2|+4\ln|x+2|+C$

11 $5\ln|x+1|-\frac{1}{x+1}-3\ln|x-5|+C$

13 $5\ln|x|-\frac{2}{x}+\frac{3}{2x^2}-\frac{1}{3x^3}+4\ln|x+3|+C$

15 $x+4\ln|x|+\ln(x^2+4)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$

17 $\ln(x^2+4)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+3\ln|x+5|+C$

19 $-\frac{1}{2}\ln(x^2+4)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{2}\ln(x^2+1)+C$

21 $\ln(x^2+1)-\frac{4}{x^2+1}+C$

23 $\frac{1}{2}x^2+x+2\ln|x|+2\ln|x-1|+C$

25 $\frac{1}{3}x^3-9x-\frac{1}{9x}-\frac{1}{2}\ln(x^2+9)+\frac{728}{27}\tan^{-1}\left(\frac{x}{3}\right)+C$

27 $2\ln|x+4|+\frac{6}{x+4}-\frac{5}{x-3}+C$

29 $\frac{13}{6}\ln(6x+5)+\frac{8}{3}\ln(3x-2)-\ln(2x+7)+4\ln(x-1)+C$

31 $-\frac{34}{5}\ln(5x+2)-\frac{17}{3}\ln(3x+25)+\frac{3}{2}\ln(2x-5)+C$

33 $\frac{1}{2a}(\ln|a+u|-\ln|a-u|)+C=\frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right|+C$

35 $-\frac{b}{a^2}\ln|u|-\frac{1}{au}+\frac{b}{a^2}\ln|a+bu|+C = -\frac{1}{au}+\frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right|+C$

37 $\frac{1}{2}\ln 3\approx 0.55$ **39** $\frac{\pi}{27}(4\ln 2+3)\approx 0.67$

41 $\frac{\frac{7}{4}}{x}+\frac{-\frac{5}{3}}{x-1}+\frac{-\frac{1}{3}}{x+1}+\frac{\frac{29}{24}}{x-2}+\frac{\frac{25}{24}}{x+2}$

Exercises 7.5

1 $\frac{1}{2}\tan^{-1}\frac{x+1}{2}+C$ **3** $\frac{1}{2}\tan^{-1}\frac{x-2}{2}+C$

5 $\sin^{-1}\frac{x-2}{2}+C$

7 $-2\sqrt{9-8x-x^2}-5\sin^{-1}\frac{x+4}{5}+C$

9 $\frac{1}{2}\left[\tan^{-1}(x+2)+\frac{x+2}{x^2+4x+5}\right]+C$

11 $\frac{x+3}{4\sqrt{x^2+6x+13}}+C$ **13** $\frac{2}{3\sqrt{7}}\tan^{-1}\frac{4x-3}{3\sqrt{7}}+C$

15 $\ln\left(\frac{e^x+1}{e^x+2}\right)+C$ **17** $1+\frac{\pi}{4}\approx 1.79$ **19** $\frac{\pi}{20}\approx 0.16$

21 $\frac{3}{7}(x+9)^{7/3}-\frac{27}{4}(x+9)^{4/3}+C$

23 $\frac{5}{81}(3x+2)^{9/5}-\frac{5}{18}(3x+2)^{4/5}+C$

25 $2+8\ln\frac{6}{7}\approx 0.767$

27 $\frac{6}{7}x^{7/6}-\frac{6}{5}x^{5/6}+2x^{1/2}-6x^{1/6}+6\tan^{-1}(x^{1/6})+C$

29 $\frac{2}{\sqrt{3}}\tan^{-1}\sqrt{\frac{x-2}{3}}+C$

31 $\frac{3}{5}(x+4)^{5/3}-\frac{9}{2}(x+4)^{2/3}+C$

33 $\frac{2}{7}(1+e^x)^{7/2}-\frac{4}{5}(1+e^x)^{5/2}+\frac{2}{3}(1+e^x)^{3/2}+C$

35 $e^x-4\ln(e^x+4)+C$

37 $2\sin\sqrt{x+4}-2\sqrt{x+4}\cos\sqrt{x+4}+C$ **39** $\frac{137}{320}$

41 $\ln|\cos x|-\ln(1-\cos x)+C$

43 $\frac{1}{2}\ln|e^x-1|-\frac{1}{2}\ln(e^x+1)+C$

45 $\frac{4}{3}\ln(4-\sin x)+\frac{2}{3}\ln(\sin x+2)+C$

47 $\frac{2}{\sqrt{3}}\tan^{-1}\frac{2\tan(x/2)+1}{\sqrt{3}}+C$ **49** $\ln\left|\tan\frac{x}{2}+1\right|+C$

51 $-\frac{1}{5}\ln\left|2\tan\frac{x}{2}-1\right|+\frac{1}{5}\ln\left|\tan\frac{x}{2}+2\right|+C$

Exercises 7.6

1 $\sqrt{4+9x^2} - 2\ln\left|\frac{2+\sqrt{4+9x^2}}{3x}\right| + C$

3 $-\frac{x}{8}(2x^2-80)\sqrt{16-x^2} + 96\sin^{-1}\frac{x}{4} + C$

5 $-\frac{2}{135}(9x+4)(2-3x)^{3/2} + C$

7 $-\frac{1}{18}\sin^5 3x\cos 3x - \frac{5}{72}\sin^3 3x\cos 3x - \frac{5}{48}\sin 3x\cos 3x + \frac{5}{16}x + C$

9 $-\frac{1}{3}\cot x\csc^2 x - \frac{2}{3}\cot x + C$

11 $\frac{2x^2-1}{4}\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + C$

13 $\frac{1}{13}e^{-3x}(-3\sin 2x - 2\cos 2x) + C$

15 $\sqrt{5x-9x^2} + \frac{5}{6}\cos^{-1}\frac{5-18x}{5} + C$

17 $\frac{1}{4\sqrt{15}}\ln\left|\frac{\sqrt{5}x^2-\sqrt{3}}{\sqrt{5}x^2+\sqrt{3}}\right| + C$

19 $\frac{1}{4}(2e^{2x}-1)\cos^{-1}e^x - \frac{1}{4}e^x\sqrt{1-e^{2x}} + C$

21 $\frac{2}{315}(35x^3-60x^2+96x-128)(2+x)^{3/2} + C$

23 $\frac{2}{81}(4+9\sin x - 4\ln|4+9\sin x|) + C$

25 $2\sqrt{9+2x} + 3\ln\left|\frac{\sqrt{9+2x}-3}{\sqrt{9+2x}+3}\right| + C$

27 $\frac{3}{4}\ln\left|\frac{\sqrt[3]{x}}{4+\sqrt[3]{x}}\right| + C$

29 $\sqrt{16-\sec^2 x} - 4\ln\left|\frac{4+\sqrt{16-\sec^2 x}}{\sec x}\right| + C$

31 $\frac{1}{2}\ln(\cos x+\sin x+1) - \frac{1}{2}\ln(5\cos x+\sin x+5) + C$

33 $e^{4x}\left[\frac{1}{5000}(1000x^3-450x^2+60x+21)\sin 2x - \frac{1}{2500}(250x^3-300x^2+165x-36)\cos 2x\right] + C$

35 $2\sqrt{x} - \ln(x+\sqrt{x}+3) - \frac{10}{11}\sqrt{11}\tan^{-1}\left(\frac{\sqrt{11}(2\sqrt{x}+1)}{11}\right) + C$

37 $\ln\left(\frac{1-\cos x}{\sin x}\right) + C$ **39** $2\sqrt{x} - 2\ln(\sqrt{x}+1) + C$

Exercises 7.7

C denotes that the integral converges; D denotes that it diverges.

1 C; 3 **3** D **5** D **7** C; $\frac{1}{2}$ **9** C; $-\frac{1}{2}$

11 D **13** D **15** C; 0 **17** D **19** D **21** C **23** D

25 **(a)** Not possible **(b)** π **27** π

29 **(b)** No **31** If $F(x) = \frac{k}{x^2}$, then $W = k$.

33 **(a)** $\frac{1}{k}$ **(b)** No, the improper integral diverges.

35 **(b)** $c = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}$ **37** $\frac{1}{s}$, $s > 0$ **39** $\frac{s}{s^2+1}$, $s > 0$

41 $\frac{1}{s-a}$, $s > a$

43 **(a)** 1; 1; 2 **(b)** *Hint:* Let $u = x^n$ and integrate by parts.

45 0.49

47 C; 6 **49** D **51** D **53** D **55** C; $3\sqrt[3]{4}$ **57** D

59 C; $\frac{\pi}{2}$ **61** D **63** C; $-\frac{1}{4}$ **65** D **67** D

69 D **71** C **73** D

75 $n > -1$ **77** **(a)** 2 **(b)** Not possible **79** 1.79

81 **(b)** $T = 2\pi\sqrt{\frac{m}{k}}$ **83** **(a)** t is undefined at $y = 0$.

Chapter 7 Review Exercises

1 $\frac{1}{2}x^2\sin^{-1}x - \frac{1}{4}\sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + C$

3 $2\ln 2 - 1 \approx 0.39$ **5** $\frac{1}{6}\sin^3 2x - \frac{1}{10}\sin^5 2x + C$

7 $\frac{1}{5}\sec^5 x + C$ **9** $\frac{x}{25\sqrt{x^2+25}} + C$

11 $2\ln\left|\frac{2-\sqrt{4-x^2}}{x}\right| + \sqrt{4-x^2} + C$

13 $2\ln|x-1| - \ln|x| - \frac{x}{(x-1)^2} + C$

15 $-5\ln|x-3| + 2\ln|x+3| + 2\ln(x^2+9) + \frac{1}{3}\tan^{-1}\frac{x}{3} + C$

17 $-\sqrt{4+4x-x^2} + 2\sin^{-1}\frac{x-2}{\sqrt{8}} + C$

19 $3(x+8)^{1/3} + \ln[(x+8)^{1/3}-2]^2 - \ln|(x+8)^{2/3}+2(x+8)^{1/3}+4| - \frac{6}{\sqrt{3}}\tan^{-1}\frac{(x+8)^{1/3}+1}{\sqrt{3}} + C$

21 $\frac{1}{13}e^{2x}(2\sin 3x - 3\cos 3x) + C$

23 $\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C$ **25** $-\sqrt{4-x^2} + C$

27 $\frac{1}{3}x^3 - x^2 + 3x - \frac{1}{4}\ln|x| - \frac{1}{2x} - \frac{23}{4}\ln|x+2| + C$

29 $2\tan^{-1}\sqrt{x} + C$ **31** $\ln|\sec e^x + \tan e^x| + C$

33 $\frac{1}{125}[10x\sin 5x - (25x^2-2)\cos 5x] + C$

35 $\frac{2}{7}\cos^{7/2}x - \frac{2}{3}\cos^{3/2}x + C$ **37** $\frac{2}{3}(1+e^x)^{3/2} + C$

39 $\frac{1}{16}[2x\sqrt{4x^2+25} - 25\ln(\sqrt{4x^2+25}+2x)] + C$

41 $\frac{1}{3}\tan^3 x + C$ **43** $-x\csc x + \ln|\csc x - \cot x| + C$

45 $-\frac{1}{4}(8-x^3)^{4/3} + C$

47 $-2x\cos\sqrt{x} + 4\sqrt{x}\sin\sqrt{x} + 4\cos\sqrt{x} + C$

49 $\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + C$

51 $\frac{2}{5}x^{5/2} - \frac{8}{3}x^{3/2} + 6x^{1/2} + C$

53 $\frac{1}{3}(16-x^2)^{3/2} - 16(16-x^2)^{1/2} + C$

55 $\frac{11}{2}\ln|x+5| - \frac{15}{2}\ln|x+7| + C$

57 $x\tan^{-1}5x - \frac{1}{10}\ln(1+25x^2) + C$ **59** $e^{\tan x} + C$

61 $\frac{1}{\sqrt{5}}\ln|\sqrt{7+5x^2} + \sqrt{5}x| + C$

63 $-\frac{1}{5}\cot^5 x + \frac{1}{3}\cot^3 x - \cot x - x + C$

65 $\frac{1}{5}(x^2-25)^{5/2} + \frac{25}{3}(x^2-25)^{3/2} + C$

67 $\frac{1}{3}x^3 - \frac{1}{4}\tanh 4x + C$

69 $-\frac{1}{4}x^2e^{-4x} - \frac{1}{8}xe^{-4x} - \frac{1}{32}e^{-4x} + C$

71 $3\sin^{-1}\frac{x+5}{6} + C$ **73** $-\frac{1}{7}\cos 7x + C$

75 $-9\ln|x-1| + 18\ln|x-2| - 5\ln|x-3| + C$

77 $x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + \sin x + C$

79 $-\frac{\sqrt{9-4x^2}}{x} - 2\sin^{-1}\left(\frac{2}{3}x\right) + C$

81 $24x - \frac{10}{3}\ln|\sin 3x| - \frac{1}{3}\cot 3x + C$

83 $-\ln x - \frac{4}{\sqrt[4]{x}} + 4\ln(\sqrt[4]{x}+1) + C$

85 $-2\sqrt{1+\cos x} + C$

87 $-\frac{x}{2(25+x^2)} + \frac{1}{10}\tan^{-1}\frac{x}{5} + C$

89 $\frac{1}{3}\sec^3 x - \sec x + C$

91 $\frac{7}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) - \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + \ln(x^2+4) + C$

93 $\frac{1}{4}x^4 - 2x^2 + 4\ln|x| + C$

95 $\frac{2}{5}x^{5/2}\ln x - \frac{4}{25}x^{5/2} + C$

97 $\frac{3}{64}(2x+3)^{8/3} - \frac{9}{20}(2x+3)^{5/3} + \frac{27}{16}(2x+3)^{2/3} + C$

99 $\frac{1}{2}e^{(x^2)}(x^2-1) + C$

101 D **103** D **105** C; $-\frac{9}{2}$ **107** D

109 C; $\frac{\pi}{2}$ **111** D **113** 0.14

115 **(a)** 1 **(b)** $\frac{\pi}{32}$

117 **(a)** Not possible **(b)** Not possible

CHAPTER ■ 8

Exercises 8.1

1 $\frac{1}{5}, \frac{1}{4}, \frac{3}{11}, \frac{2}{7}; \frac{1}{3}$ **3** $\frac{3}{5}, -\frac{9}{11}, -\frac{29}{21}, -\frac{57}{35}; -2$

5 $-5, -5, -5, -5; -5$ **7** $2, \frac{7}{3}, \frac{25}{14}, \frac{7}{5}; 0$

9 $\frac{2}{\sqrt{10}}, \frac{2}{\sqrt{13}}, \frac{2}{\sqrt{18}}, \frac{2}{5}; 0$ **11** $\frac{3}{10}, -\frac{6}{17}, \frac{9}{26}, -\frac{12}{37}; 0$

13 1.1, 1.01, 1.001, 1.0001; 1 **15** 2, 0, 2, 0; DNE

17 C; 0 **19** C; $\frac{\pi}{2}$ **21** D **23** C; 0 **25** D **27** D

29 C; e **31** C; 0 **33** C; $\frac{1}{2}$ **35** D **37** C; 1

39 C; 0 **41** C; 0

43 **(b)** 10,000 on A; 5000 on B; 20,000 on C

45 **(a)** The sequence appears to converge to 1.
(b) Use mathematical induction; 1

47 **(a)** The sequence appears to converge to approximately 0.739.

49 **(a)** $x_2 = 3.5$, $x_3 = 3.178571429$, $x_4 = 3.162319422$, $x_5 = 3.162277660$, $x_6 = 3.162277660$

51 **(a)** $B = \frac{1}{4}$ **(b)** 1.10

Exercises 8.2

1 **(a)** $-\frac{2}{35}, -\frac{4}{45}, -\frac{6}{55}$ **(b)** $-\frac{2n}{5(2n+5)}$ **(c)** C; $-\frac{1}{5}$

3 **(a)** $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$ **(b)** $\frac{n}{2n+1}$ **(c)** C; $\frac{1}{2}$

5 **(a)** $-\ln 2, -\ln 3, -\ln 4$ **(b)** $-\ln(n+1)$ **(c)** D

7 C; 4 **9** C; $\frac{\sqrt{5}}{\sqrt{5}+1}$ **11** C; $\frac{37}{99}$ **13** D **15** D

17 $-1 < x < 1$; $\frac{1}{1+x}$ **19** $1 < x < 5$; $\frac{1}{5-x}$ **21** $\frac{23}{99}$

23 $\frac{16{,}181}{4995}$ **25** C **27** C **29** D **31** D **33** D

35 Needs further investigation **37** D **39** D

41 C; $\frac{41}{24}$ **43** C; $\frac{6}{7}$ **45** C; $\frac{8}{7}$ **47** C; $\frac{5}{3}$

49 **(a)** 0.21037; 0.26720; 0.26940 **(b)** 0.265

51 $S_{32} \approx 4.06$

53 1.423611; 1.527422; 1.564977; 1.584347; 1.596163

55 1.040293; 1.573514; 1.921645; 2.179883; 2.385110

57 Disprove; let $a_n = 1$ and $b_n = -1$ **59** 30 m

61 (b) $\dfrac{Q}{1 - e^{-cT}}$ **(c)** $-\dfrac{1}{c}\ln\dfrac{M - Q}{M}$ **63 (b)** 2000

65 (a) $a_{k+1} = \frac{1}{4}\sqrt{10}\,a_k$

(b) $a_n = \left(\frac{1}{4}\sqrt{10}\right)^{n-1} a_1$; $A_n = \left(\frac{5}{8}\right)^{n-1} A_1$;

$P_n = \left(\frac{1}{4}\sqrt{10}\right)^{n-1} P_1$

(c) $\dfrac{16}{4 - \sqrt{10}}a_1$; $\frac{8}{3}a_1^2$

Exercises 8.3

Exer. 1–12: (a) Each function f is positive-valued and continuous on the interval of integration. Since $f'(x)$ is negative, f is decreasing. (b) The value of the improper integral is given, if it exists.

1 (a) $f'(x) = \dfrac{-4}{(2x + 3)^3} < 0$ if $x \geq 1$

(b) $\int_1^\infty f(x)\,dx = \frac{1}{10}$; C

3 (a) $f'(x) = \dfrac{-4}{(4x + 7)^2} < 0$ if $x \geq 1$

(b) $\int_1^\infty f(x)\,dx = \infty$; D

5 (a) $f'(x) = x(2 - 3x^3)e^{-x^3} < 0$ if $x \geq 1$

(b) $\int_1^\infty f(x)\,dx = \frac{1}{3e}$; C

7 (a) $f'(x) = \dfrac{1 - \ln x}{x^2} < 0$ if $x \geq 3$

(b) $\int_3^\infty f(x)\,dx = \infty$; D

9 (a) $f'(x) = \dfrac{1 - 2x^2}{x^2(x^2 - 1)^{3/2}} < 0$ if $x \geq 2$

(b) $\int_2^\infty f(x)\,dx = \frac{\pi}{6}$; C

11 (a) $f'(x) = \dfrac{1 - 2x \arctan x}{(1 + x^2)^2} < 0$ if $x \geq 1$

(b) $\int_1^\infty f(x)\,dx = \dfrac{3\pi^2}{32}$; C

Exer. 13–28: A typical b_n is listed; however, there are many other possible choices.

13 $b_n = \frac{1}{n^4}$; C **15** $b_n = \frac{1}{3^n}$; C **17** $b_n = \frac{\pi/4}{n}$; D

19 $b_n = \frac{1}{n^2}$; C **21** $b_n = \frac{1}{\sqrt{n}}$; D **23** $b_n = \frac{1}{n^{3/2}}$; C

25 $b_n = \frac{1}{e^n}$; C **27** $b_n = \frac{1}{\sqrt{n}}$; D **29** D **31** C

33 D **35** D **37** C **39** C **41** C **43** C

45 C **47** $k > 1$ **49 (b)** $n > e^{100} - 1 \approx 2.688 \times 10^{43}$

51 Since $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, there is an M such that if $K > M$, then $\frac{a_k}{b_k} < 1$, or $a_k < b_k$. Since Σb_n converges and $a_n < b_n$ for all but at most a finite number of terms, Σa_n must also converge.

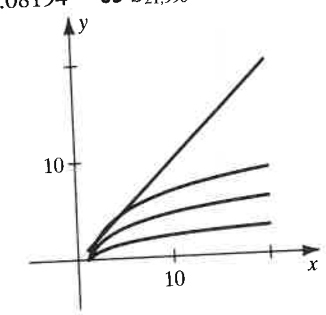
53 $\sum_{k=1}^\infty a_k = \sum_{k=1}^n a_k + \sum_{k=n+1}^\infty a_k$, where the error $E = \sum_{k=n+1}^\infty a_k < \int_n^\infty f(x)\,dx$. (See Figure 8.8.)

55 4

57 Since Σa_n converges, $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} \frac{1}{a_n} = \infty$. By (8.17), $\Sigma \frac{1}{a_n}$ diverges.

59 $S_3 \approx 0.40488$ **61** $S_9 \approx 1.08194$ **63** $S_{21{,}998} \approx 0.93705$

65 The series diverges for $k = 1$, 2, and 3.



Exercises 8.4

1 $\frac{1}{2}$; C **3** $\frac{5}{3}$; D **5** 0; C **7** 1; inconclusive

9 ∞; D **11** 0; C **13** 2; D **15** $\frac{1}{3}$; C **17** $\frac{1}{2}$; C

19 C **21** C **23** C **25** C **27** D **29** C

31 D **33** C **35** D **37** D **39** D

Exercises 8.5

1 (a) Conditions (i) and (ii) are satisfied.
(b) Converges, by (8.30)

3 (a) Condition (i) is satisfied, but (ii) is not.
(b) Diverges, by (8.17)

5 CC **7** CC **9** D **11** AC **13** AC **15** D **17** CC

19 AC **21** D **23** CC **25** D **27** D **29** D **31** AC

33 0.368 **35** 0.901 **37** 0.306 **39** 141 **41** 5

45 No. If $a_n = b_n = \frac{(-1)^n}{\sqrt{n}}$, then both Σa_n and Σb_n converge by the alternating series test. However, $\Sigma a_n b_n = \Sigma \frac{1}{n}$, which diverges.

Exercises 8.6

1 $[-1, 1)$ **3** $(-2, 2)$ **5** $(-1, 1]$ **7** $[-1, 1)$

9 $[-1, 1]$ **11** $(-6, 14)$ **13** Converges only for $x = 0$

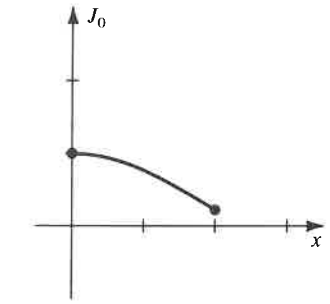
15 $(-2, 2)$ **17** $(-\infty, \infty)$ **19** $\left[\frac{17}{9}, \frac{19}{9}\right)$ **21** $(-12, 4)$

23 Converges only for $x = 3$ **25** $(0, 2e)$ **27** $\left(-\frac{5}{2}, \frac{7}{2}\right]$

29 $(-\infty, \infty)$

31 (a) $\frac{3}{2}$ **33 (a)** $\frac{1}{e}$ **35** ∞ **37** Use (8.35).

39 $J_0(x) \approx 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304}$ **41** Use (8.35). **43** Use (8.37).



45 Assume that $\Sigma a_n x^n$ is absolutely convergent at $x = r$. Let $x = -r$. Then $\Sigma|a_n(-r)^n| = \Sigma|a_n r^n|$ is absolutely convergent, which implies that $\Sigma a_n(-r)^n$ is convergent. This is a contradiction.

Exercises 8.7

1 (a) $\sum_{n=0}^\infty 3^n x^n$ **(b)** $\sum_{n=1}^\infty n3^n x^{n-1}$; $\sum_{n=0}^\infty \frac{3^n}{n+1}x^{n+1}$

3 (a) $\frac{1}{2}\sum_{n=0}^\infty (-1)^n \left(\frac{7}{2}\right)^n x^n$

(b) $\frac{1}{2}\sum_{n=1}^\infty (-1)^n \frac{n7^n}{2^n}x^{n-1}$; $\frac{1}{2}\sum_{n=0}^\infty (-1)^n \frac{7^n}{(n+1)2^n}x^{n+1}$

5 $\sum_{n=0}^\infty x^{2n+2}$; $r = 1$ **7** $\sum_{n=0}^\infty \frac{3^n}{2^{n+1}}x^{n+1}$; $r = \frac{2}{3}$

9 $-1 - x - 2\sum_{n=2}^\infty x^n$; $r = 1$ **11 (b)** 0.183; 0.182321557

15 $\sum_{n=0}^\infty \frac{3^n}{n!}x^{n+1}$ **17** $\sum_{n=0}^\infty (-1)^n \frac{1}{n!}x^{n+3}$

19 $\sum_{n=0}^\infty (-1)^n \frac{1}{n+1}x^{2n+4}$ **21** $\sum_{n=0}^\infty (-1)^n \frac{1}{2n+1}x^{(2n+1)/2}$

23 $\sum_{n=0}^\infty \frac{-5^{2n+1}}{(2n+1)!}x^{2n+1}$ **25** $\sum_{n=0}^\infty \frac{1}{(2n)!}x^{6n+2}$ **27** 0.3333

29 0.0992 **31** 0.9677 **33** $\sum_{n=1}^\infty (2n)x^{2n-1}$ **37** $-\sum_{n=1}^\infty \frac{1}{n^2}x^n$

39 (a) $\sum_{n=0}^\infty (-1)^n \frac{x^{4n+1}}{4n+1}$ **41 (a)** $\sum_{n=0}^\infty (-1)^n \frac{x^{n+1}}{(n+1)(n+1)!}$

Exercises 8.8

1 $\frac{3^n}{n!}$

3 $a_n = 0$ if $n = 2k$, and $a_n = (-1)^k \frac{2^{2k+1}}{(2k+1)!}$ if $n = 2k + 1$

5 $(-1)^n 3^n$ **7 (b)** $\sum_{n=0}^\infty (-1)^n \frac{1}{(2n)!}x^{2n}$

9 $\sum_{n=0}^\infty (-1)^n \frac{3^{2n+1}}{(2n+1)!}x^{2n+2}$ **11** $\sum_{n=0}^\infty (-1)^n \frac{2^{2n}}{(2n)!}x^{2n}$

13 $1 + \sum_{n=1}^\infty (-1)^n \frac{2^{2n-1}}{(2n)!}x^{2n}$ **15** $\sum_{n=0}^\infty \frac{(\ln 10)^n}{n!}x^n$

17 $\sum_{n=0}^\infty (-1)^n \frac{1}{\sqrt{2}(2n+1)!}\left(x - \frac{\pi}{4}\right)^{2n+1} +$

$\sum_{n=0}^\infty (-1)^n \frac{1}{\sqrt{2}(2n)!}\left(x - \frac{\pi}{4}\right)^{2n}$

19 $\sum_{n=0}^\infty (-1)^n \frac{1}{2^{n+1}}(x - 2)^n$ **21** $\sum_{n=0}^\infty \frac{2^n}{e^2 n!}(x + 1)^n$

23 $2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 7\left(x - \frac{\pi}{3}\right)^2$

25 $\frac{\pi}{6} + \frac{2}{\sqrt{3}}\left(x - \frac{1}{2}\right) + \frac{2}{3\sqrt{3}}\left(x - \frac{1}{2}\right)^2$

27 $-\frac{1}{e} + \frac{1}{2e}(x + 1)^2 + \frac{1}{3e}(x + 1)^3$ **29** 0.5; 0.125

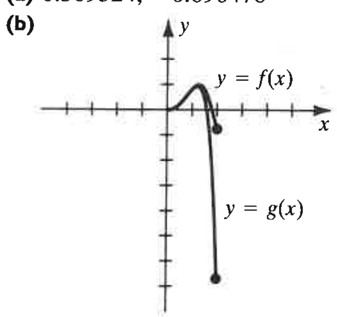
31 0.9986; 3.13×10^{-7} **33** 0.0997; 2×10^{-6}

35 0.6667; 0.1 **37** 0.4969; 9.04×10^{-6} **39** 0.4864

41 0.4484

43 (a) 0.309524, −0.690476

(b)



The first approximation is more accurate.

45 $2\sum_{n=0}^\infty \frac{1}{2n+1}x^{2n+1}$

47 (a) $\pi = 4\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + (-1)^n \frac{1}{2n+1} + \cdots\right]$

(b) 3.34 with an error of less than $\frac{4}{11}$ **(c)** 40,000

49 (c) 16.7 ft

53 (a) $\sum_{n=1}^\infty (-1)^{n+1} \frac{(3x/5)^{2n-1}}{(2n-1)!}$ **55 (a)** $\sum_{n=0}^\infty (-1)^n \frac{x^{4n}}{(2n)!}$

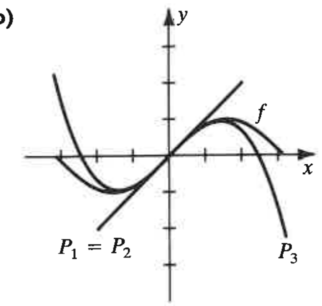
57 (a) $\sum_{n=0}^\infty (-1)^n \frac{x^{2n+1}}{(2n+1)(2n+1)!}$

59 (a) $\sum_{n=1}^\infty (-1)^{n+1} \frac{x^{6n-2}}{(6n-2)(2n-1)!}$

Exercises 8.9

1 (a) x; x; $x - \frac{1}{6}x^3$

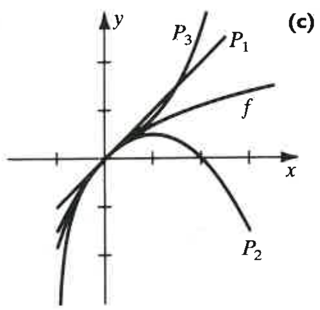
(b)



(c) 0.0500; 2.6×10^{-7}

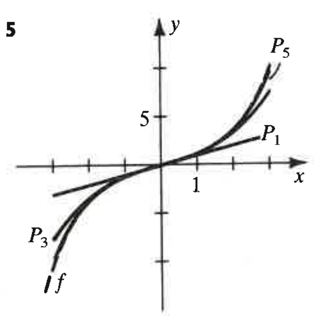
3 (a) x; $x - \frac{1}{2}x^2$; $x - \frac{1}{2}x^2 + \frac{1}{3}x^3$

(b)



(c) 0.7380; 0.164

5



$P_1(x) = x$;

$P_3(x) = x + \frac{1}{6}x^3$;

$P_5(x) = x + \frac{1}{6}x^3 + \frac{1}{120}x^5$

7 $\sin x = 1 - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 + \frac{1}{24}\sin z\left(x - \frac{\pi}{2}\right)^4$, z is between x and $\frac{\pi}{2}$.

9 $\sqrt{x} = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3 - \frac{5}{128}z^{-7/2}(x-4)^4$, z is between x and 4.

11 $\tan x = 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{1}{3}(3\tan^4 z + 4\tan^2 z + 1)\left(x - \frac{\pi}{4}\right)^3$, z is between x and $\frac{\pi}{4}$.

13 $\frac{1}{x} = -\frac{1}{2} - \frac{1}{4}(x+2) - \frac{1}{8}(x+2)^2 - \frac{1}{16}(x+2)^3 - \frac{1}{32}(x+2)^4 - \frac{1}{64}(x+2)^5 + z^{-7}(x+2)^6$, z is between x and -2.

15 $\tan^{-1} x = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{3z^2-1}{3(1+z^2)^3}(x-1)^3$, z is between x and 1.

17 $xe^x = -\frac{1}{e} + \frac{1}{2e}(x+1)^2 + \frac{1}{3e}(x+1)^3 + \frac{1}{8e}(x+1)^4 + \frac{ze^z + 5e^z}{120}(x+1)^5$, z is between x and -1.

Exer. 19–30: Since $c = 0$, z is between x and 0.

19 $\ln(x+1) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{x^5}{5(z+1)^5}$

21 $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{\sin z}{9!}x^9$

23 $e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \frac{4}{45}e^{2z}x^6$

25 $\frac{1}{(x-1)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6(z-1)^{-8}$

27 $\arcsin x = x + \frac{1+2z^2}{6(1-z^2)^{5/2}}x^3$ **29** $f(x) = -5x^3 + 2x^4$

31 0.9998; $|R_3(x)| < 4 \times 10^{-9}$

33 2.0075; $|R_3(x)| < 3 \times 10^{-10}$

35 -0.454545; $|R_5(x)| \le 5 \times 10^{-7}$

37 0.223; $|R_4(x)| < 2 \times 10^{-4}$

39 0.8660254; $|R_8(x)| < 8.2 \times 10^{-9}$

41 Five decimal places, since $|R_3(x)| \le 4.2 \times 10^{-6} < 0.5 \times 10^{-5}$

43 Three decimal places, since $|R_2(x)| \le 1.85 \times 10^{-4} < 0.5 \times 10^{-3}$

45 Four decimal places, since $|R_3(x)| \le 3.82 \times 10^{-5} < 0.5 \times 10^{-4}$

47 If f is a polynomial of degree n, then the Taylor remainder $R_n(x) = 0$, since $f^{(n+1)}(x) = 0$. By (8.45), we have $f(x) = P_n(x)$.

Chapter 8 Review Exercises

1 C; 0 **3** D **5** C; 5 **7** The terms approach 0.589388.
9 D **11** AC **13** D **15** D **17** AC **19** D
21 D **23** AC **25** CC **27** C **29** C **31** C
33 CC **35** C **37** C **39** D **41** 0.158
43 $S_4 \approx 0.63092$ **45** $(-3, 3)$ **47** $[-12, -8)$ **49** $\frac{1}{4}$

51 $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n)!}x^{2n-1}$; ∞

53 $\sum_{n=0}^{\infty}(-1)^n\frac{2^{2n}}{(2n+1)!}x^{2n+1}$; ∞ **55** (a) $\sum_{n=1}^{\infty}\frac{(3x/5)^{2n-1}}{(2n-1)!}$

57 (a) $\sum_{n=0}^{\infty}\frac{x^{4n+1}}{(4n+1)(2n)!}$ **59** $e^{-x} = e^2\sum_{n=0}^{\infty}(-1)^n\frac{1}{n!}(x+2)^n$

61 0.189 **63** 0.621

65 $\ln\cos x = \ln\left(\frac{1}{2}\sqrt{3}\right) - \frac{1}{3}\sqrt{3}\left(x - \frac{\pi}{6}\right) - \frac{2}{3}\left(x - \frac{\pi}{6}\right)^2 - \frac{4}{27}\sqrt{3}\left(x - \frac{\pi}{6}\right)^3 - \frac{1}{12}(\sec^4 z + 2\sec^2 z\tan^2 z)\left(x - \frac{\pi}{6}\right)^4$, z is between x and $\frac{\pi}{6}$.

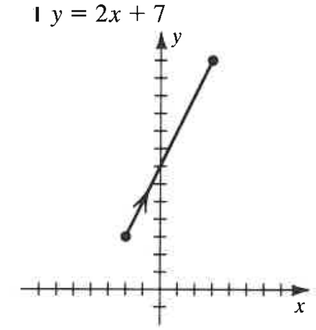
67 $e^{-x^2} = 1 - x^2 + \frac{1}{6}(4z^4 - 12z^2 + 3)e^{-z^2}x^4$, z is between x and 0.

69 0.7314

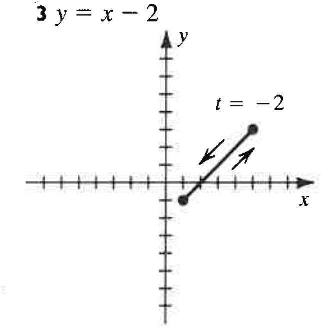
CHAPTER 9

Exercises 9.1

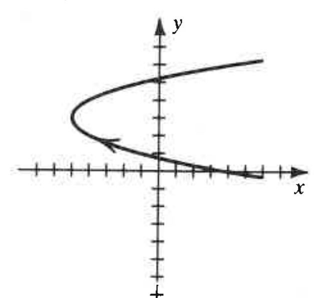
1 $y = 2x + 7$



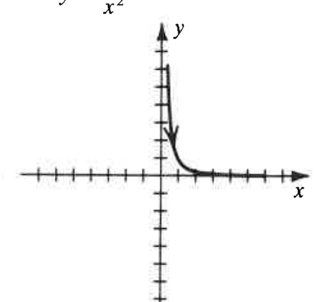
3 $y = x - 2$



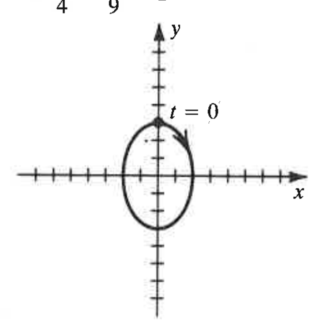
5 $(y-3)^2 = x + 5$



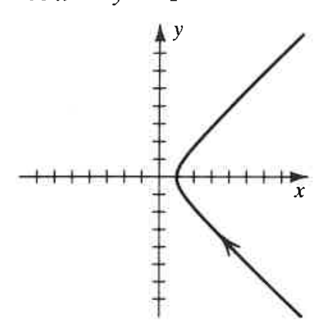
7 $y = \frac{1}{x^2}$



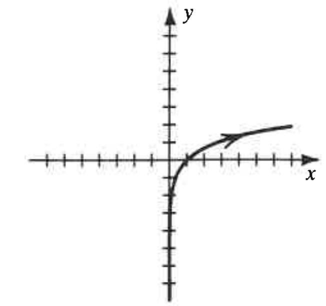
9 $\frac{x^2}{4} + \frac{y^2}{9} = 1$



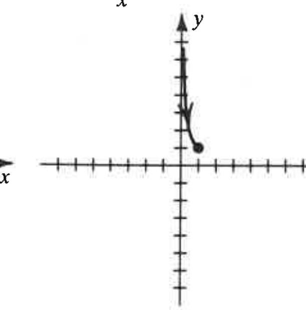
11 $x^2 - y^2 = 1$



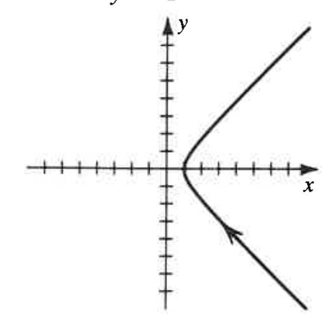
13 $y = \ln x$



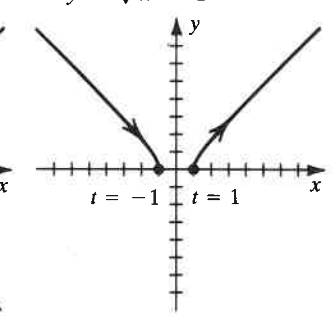
15 $y = \frac{1}{x}$



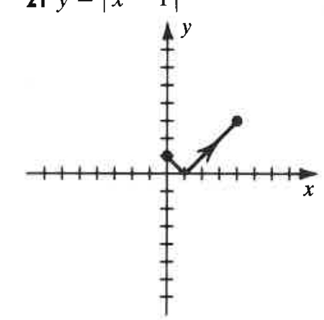
17 $x^2 - y^2 = 1$



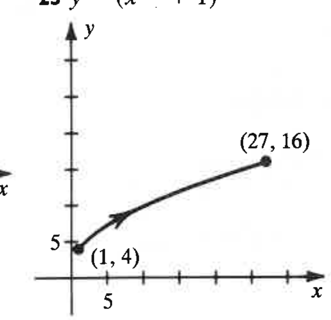
19 $y = \sqrt{x^2 - 1}$



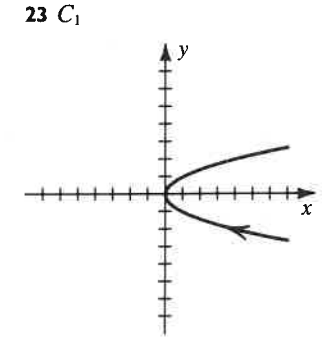
21 $y = |x - 1|$



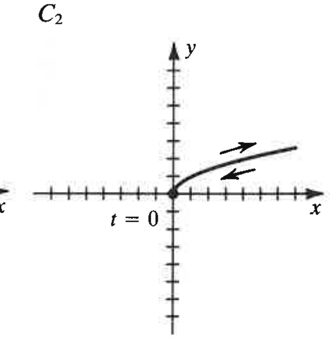
23 $y = (x^{1/3} + 1)^2$



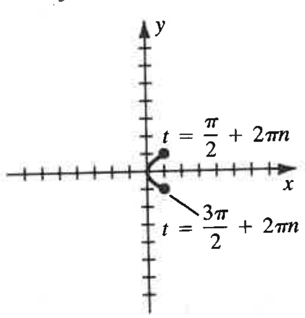
23 C_1



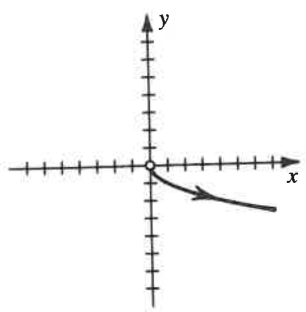
C_2



23 C_3



C_4



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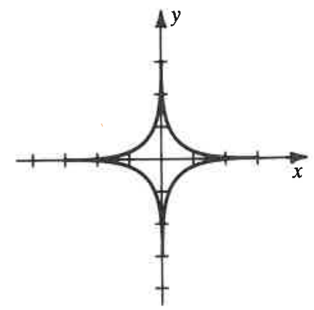
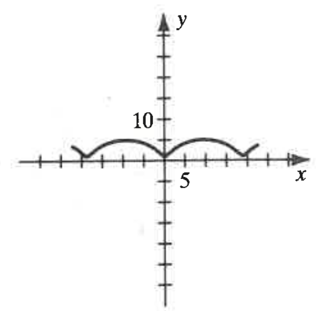
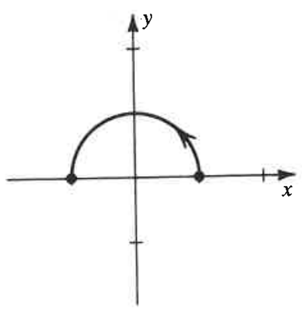


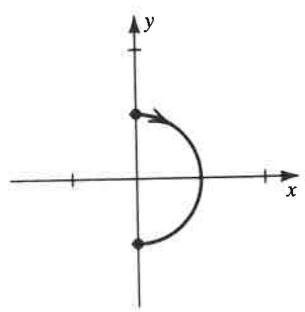
43



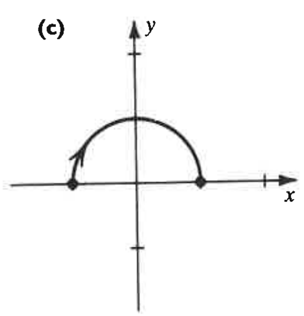
27 (a)



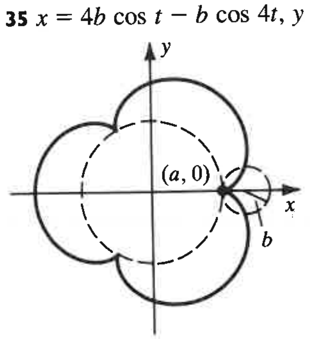
(b)



(c)

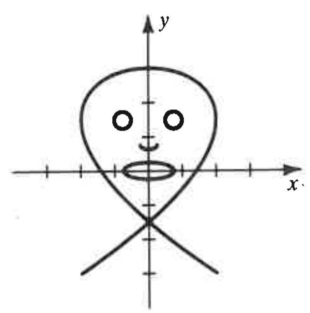


35 $x = 4b \cos t - b \cos 4t$, $y = 4b \sin t - b \sin 4t$

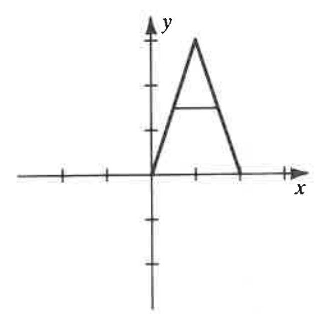


39 (a) The figure is an ellipse with center (0, 0) and axes of lengths $2a$ and $2b$.

45 A mask with a mouth, nose, and eyes



47 The letter A



49 270 ft; yes

59 Try using: $P_0(50, 35)$, $P_1(55, 49)$, $P_2(15, 49)$, $P_3(35, 25)$ (repeated), $P_4(60, -5)$, $P_5(15, -5)$, $P_6(20, 15)$

Exercises 9.2

1 1; -1 **3** $\frac{1}{4}$; -4 **5** $-\frac{2}{e^3}$; $\frac{1}{2}e^3$

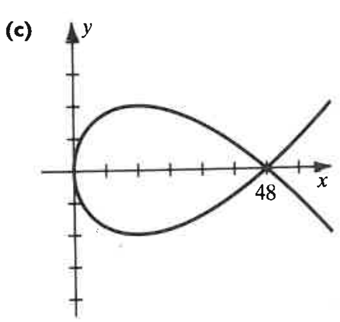
7 $-\frac{3}{2}\tan 1 \approx -2.34$; $\frac{2}{3}\cot 1 \approx 0.43$

9 $(-27, -108)$, $(1, 12)$

11 (a) Horizontal: $(16, \pm 16)$; vertical: $(0, 0)$

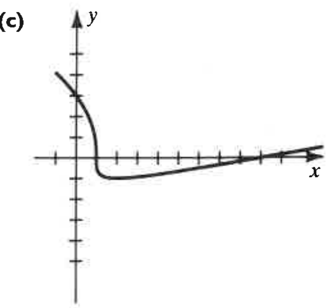
(b) $\dfrac{3t^2 + 12}{64t^3}$

(c)



13 (a) Horizontal; $(2, -1)$; vertical: $(1, 0)$ **(b)** $\dfrac{-2t + 4}{9t^5}$

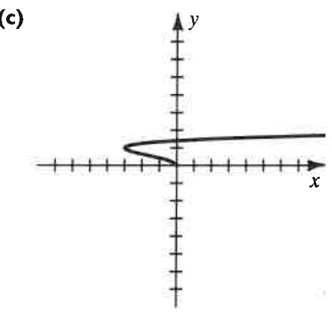
(c)



15 (a) Horizontal: none; vertical: $(0, 0)$, $(-3, 1)$

(b) $\dfrac{1 - 3t}{144t^{3/2}(t - 1)^3}$

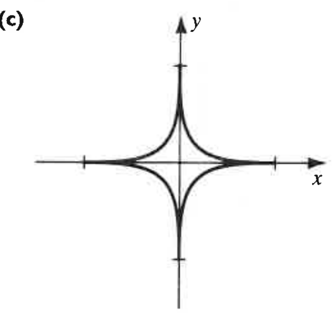
(c)



17 (a) Horizontal: $(\pm 1, 0)$; vertical: $(0, \pm 1)$

(b) $\frac{1}{3}\sec^4 t \csc t$

(c)



19 Horizontal: $(0, \pm 2)$, $(2\sqrt{3}, \pm 2)$, $(-2\sqrt{3}, \pm 2)$; vertical: $(4, \pm\sqrt{2})$, $(-4, \pm\sqrt{2})$

21 $\frac{2}{27}(34^{3/2} - 125) \approx 5.43$ **23** $\sqrt{2}(e^{\pi/2} - 1) \approx 5.39$

25 $\frac{1}{8}\pi^2 \approx 1.23$ **27** 15.9 **29** $\frac{8\pi}{3}(17^{3/2} - 1) \approx 578.83$

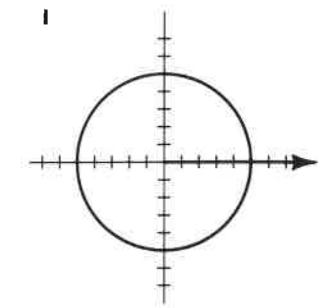
31 $\frac{11\pi}{9} \approx 3.84$ **33** $\frac{64\pi}{3} \approx 67.02$ **35** $\frac{536\pi}{5} \approx 336.78$

37 $\frac{2}{5}\sqrt{2}\pi(2e^{\pi} + 1) \approx 84.03$ **39** 2.2

43 Arc length: 142.29; segments: 203.7

Exercises 9.3

1



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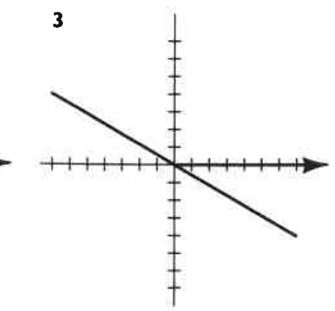

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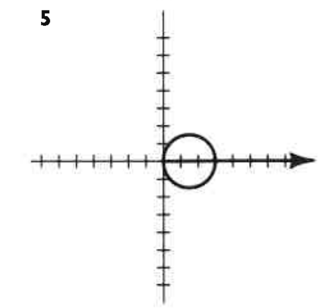

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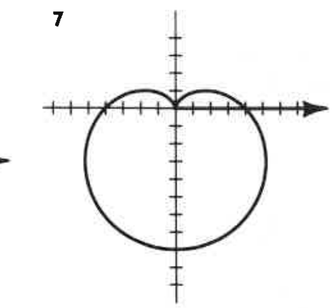

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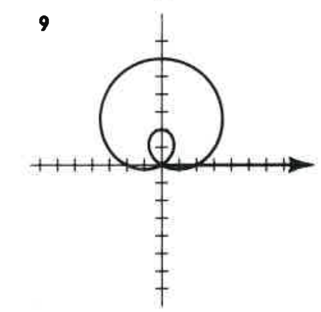

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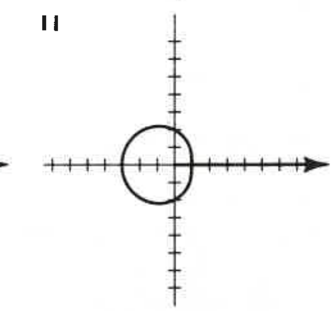

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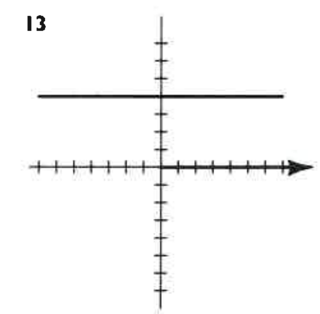

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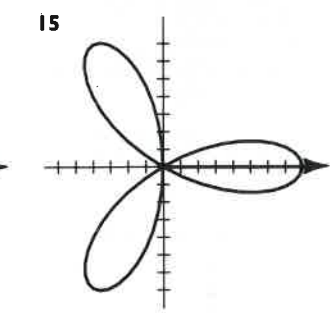

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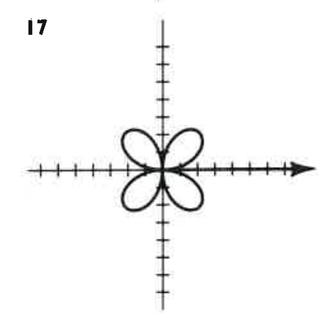

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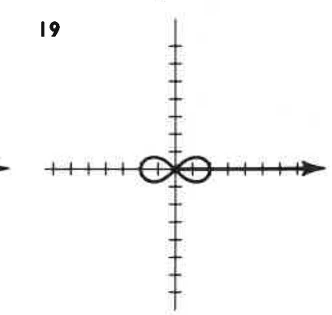

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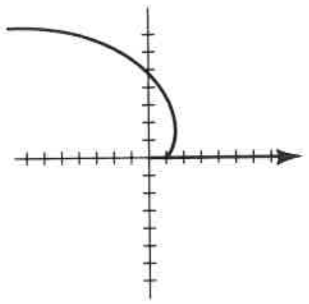

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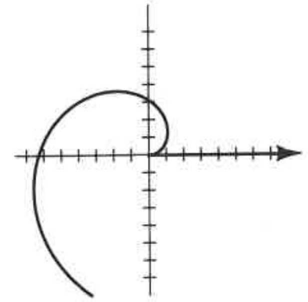

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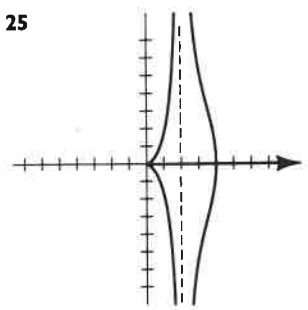


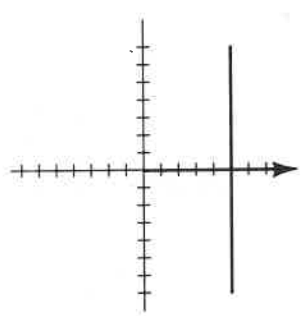
27 $r = -3 \sec \theta$ **29** $r = 4$

31 $\theta = \tan^{-1}\left(-\frac{1}{2}\right)$

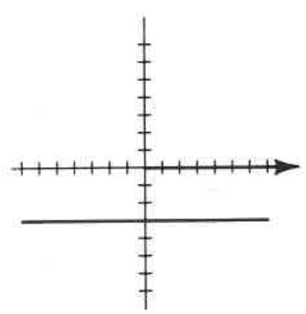
33 $r^2 = -4 \sec 2\theta$

35 $r\theta = a \sin \theta$

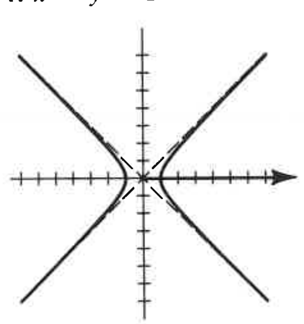
37 $x = 5$



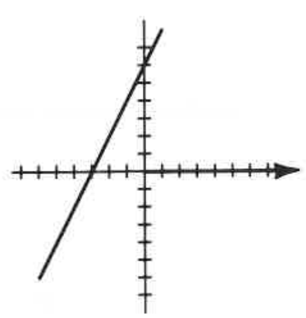
39 $y = -3$



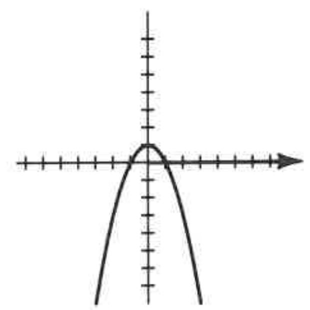
41 $x^2 - y^2 = 1$



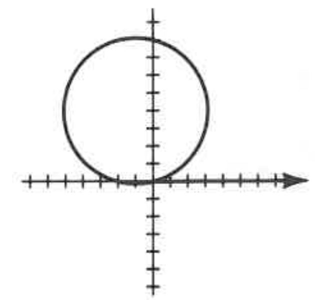
43 $y - 2x = 6$



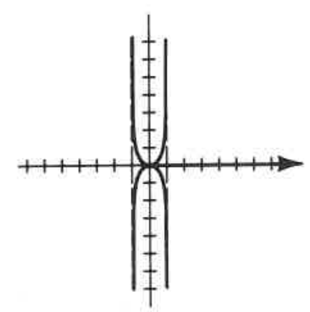
45 $y = -x^2 + 1$



47 $(x+1)^2 + (y-4)^2 = 17$



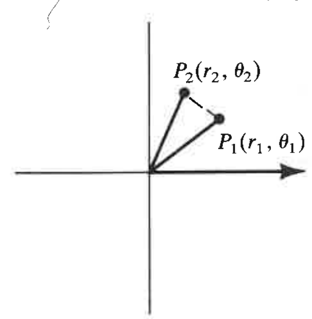
49 $y^2 = \frac{x^4}{1 - x^2}$



51 $\sqrt{3}/3$ **53** -1

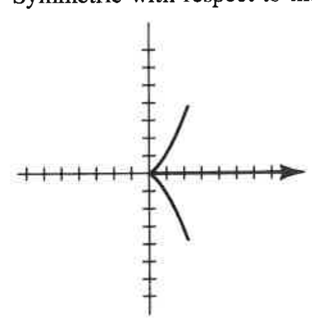
55 2 **57** 0 **59** $\frac{1}{\ln 2}$

61 Let $P_1(r_1, \theta_1)$ and $P_2(r_2, \theta_2)$ be points in an $r\theta$-plane. Let $a = r_1$, $b = r_2$, $c = d(P_1, P_2)$, and $\gamma = \theta_2 - \theta_1$. Substituting into the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos \gamma$, gives us the formula.

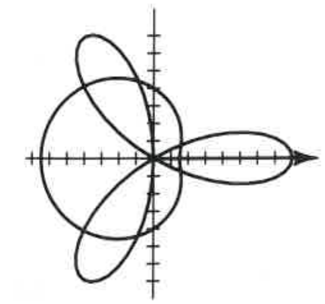


65 Use (9.11).

67 Symmetric with respect to the polar axis



69 The approximate polar coordinates are $(1.75, \pm 0.45)$, $(4.49, \pm 1.77)$, and $(5.76, \pm 2.35)$.



Exercises 9.4

1 π **3** $\frac{3\pi}{2}$ **5** $\frac{\pi}{2}$ **7** $\frac{1}{4}(e^{\pi} - 1) \approx 5.54$ **9** 2

11 $\frac{9\pi}{20}$ **13** $\int_0^{\arctan(3/4)} \frac{1}{2}(4 \sec \theta)^2\, d\theta + \int_{\arctan(3/4)}^{\pi/2} \frac{1}{2}(5)^2\, d\theta$

15 $\int_{\pi/4}^{\arctan 3} \frac{1}{2}[(4 \csc \theta)^2 - (2)^2]\, d\theta$

17 (a) $8\int_0^{\pi/6} \frac{1}{2}[(4 \cos 2\theta)^2 - (2)^2]\, d\theta$

(b) $8\left[\int_0^{\pi/6} \frac{1}{2}(2)^2\, d\theta + \int_{\pi/6}^{\pi/4} \frac{1}{2}(4 \cos 2\theta)^2\, d\theta\right]$

19 $2\pi + \frac{9}{2}\sqrt{3} \approx 14.08$ **21** $4\sqrt{3} - \frac{4\pi}{3} \approx 2.74$

23 $\frac{5\pi}{24} - \frac{1}{4}\sqrt{3} \approx 0.22$

25 $\frac{3\pi}{4} + 11 \arcsin\frac{1}{4} - \frac{1}{4}\sqrt{15} \approx 4.17$

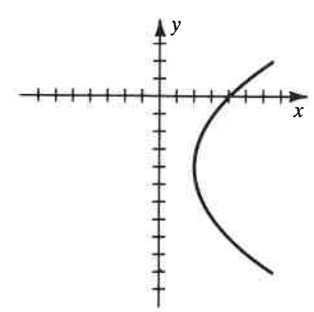
27 $\sqrt{2}(1 - e^{-2\pi}) \approx 1.41$ **29** 2 **31** $\frac{3\pi}{2}$ **33** 2.4

35 $\frac{128\pi}{5} \approx 80.42$ **37** $4\pi^2 a^2$ **39** 4.2 **41** $4\pi^2 ab$

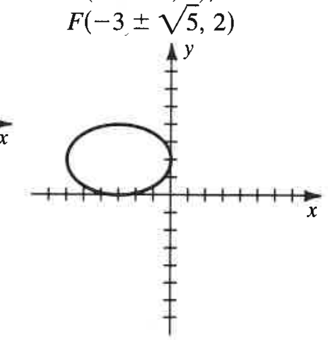
43 $\frac{2}{5}\pi\sqrt{2}\,(2 + e^{-\pi}) \approx 3.63$

Exercises 9.5

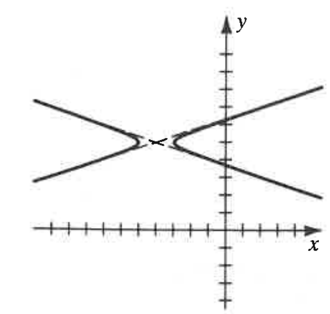
1 $V(2, -4)$; $F(4, -4)$



3 $V(-3 \pm 3, 2)$; $F(-3 \pm \sqrt{5}, 2)$

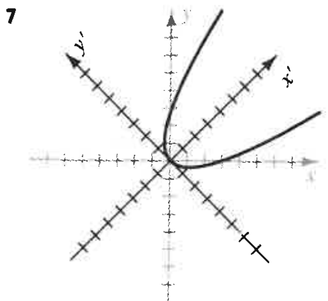


5 $V(-4 \pm 1, 5)$; $F\left(-4 \pm \frac{1}{3}\sqrt{10}, 5\right)$

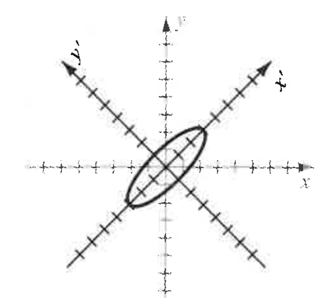


Exer. 7–19: The answer in part (a) gives the value of $B^2 - 4AC$ in the identification theorem.

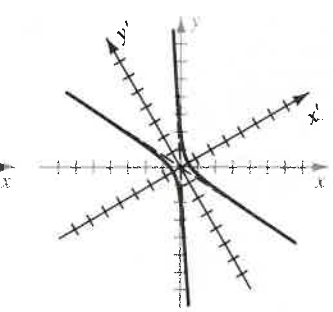
7 **(a)** 0, parabola **(b)** $(y')^2 = 2(x')$



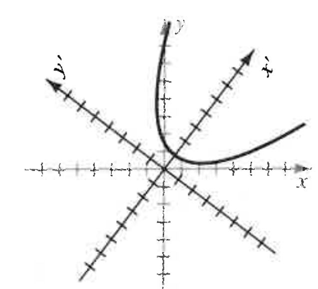
9 (a) -36, ellipse **(b)** $\frac{(x')^2}{9} + (y')^2 = 1$



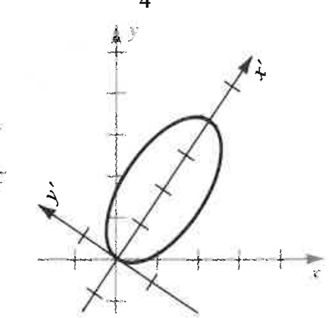
11 (a) 256, hyperbola **(b)** $\frac{(x')^2}{1/4} - (y')^2 = 1$



13 (a) 0, parabola **(b)** $(y')^2 = 4(x' - 1)$

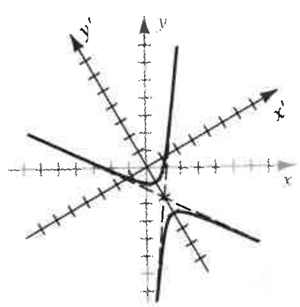


15 (a) -2704, ellipse **(b)** $\frac{(x' - 2)^2}{4} + (y')^2 = 1$



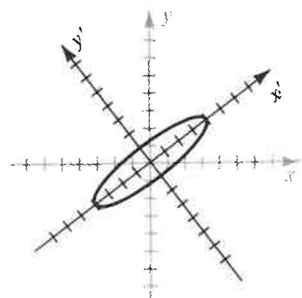
17 (a) 128, hyperbola

(b) $(y' + 2)^2 - \dfrac{(x')^2}{1/2} = 1$

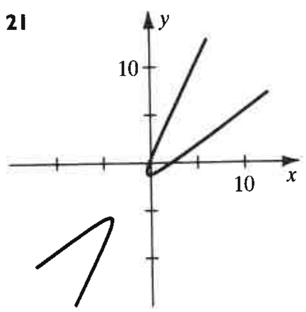


19 (a) -1600, ellipse

(b) $\dfrac{(x')^2}{16} + (y')^2 = 1$



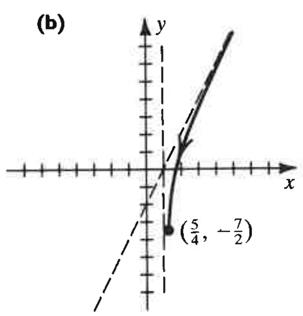
21



Chapter 9 Review Exercises

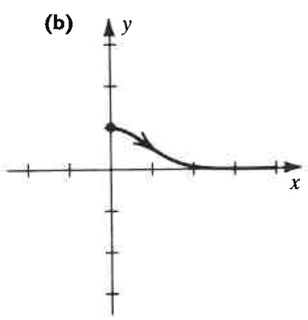
1 (a) $y = \dfrac{2x^2 - 4x + 1}{x - 1}$

(b)

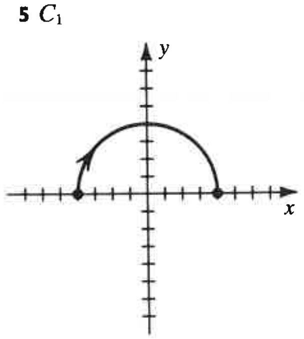


3 (a) $y = 2^{-x^2}$

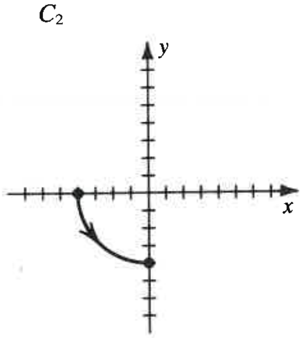
(b)



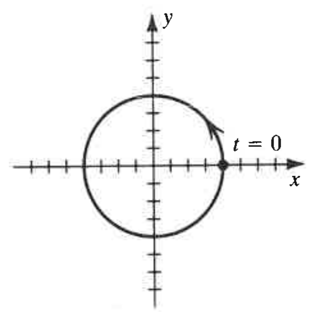
5 C_1



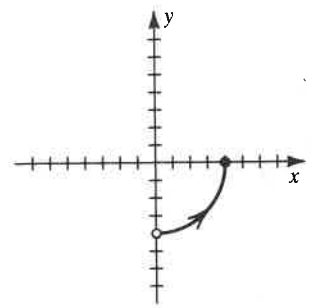
C_2



C_3



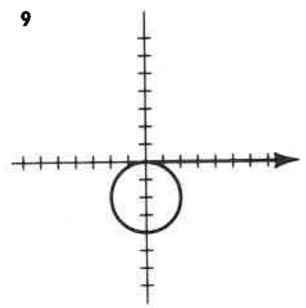
C_4



7 (a) $\dfrac{3t^2 + 2}{t}$ **(b)** Horizontal: none; vertical: 0

(c) $\dfrac{3t^2 - 2}{2t^3}$

9



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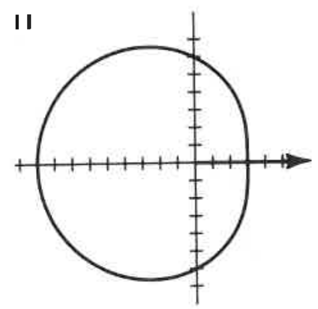

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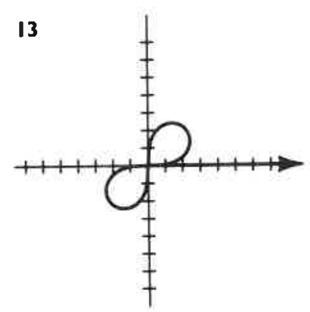

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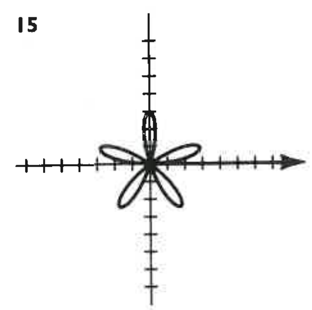

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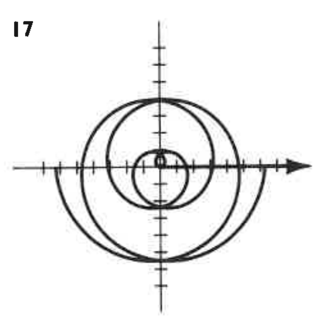

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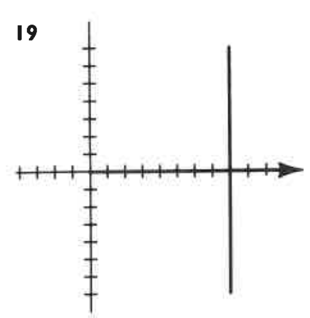

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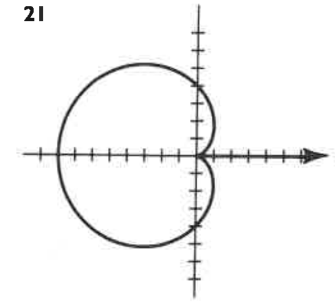

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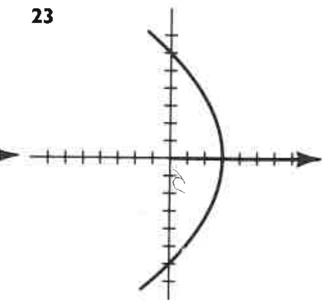

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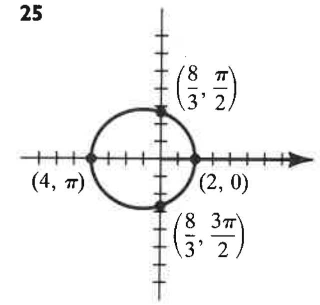


27 $r = 4 \cot \theta \csc \theta$

29 $r(2 \cos \theta - 3 \sin \theta) = 8$

31 $r = 2 \cos \theta \sec 2\theta$

33 $x^3 + xy^2 = y$

35 $(x^2 + y^2)^2 = 8xy$

37 $y = (\tan \sqrt{3})x$

39 -1

41 2

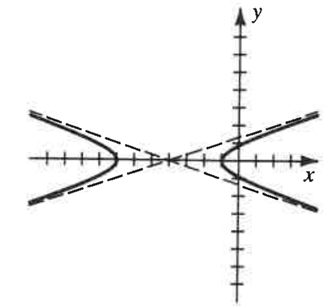
43 $\sqrt{2} + \ln(1 + \sqrt{2}) \approx 2.30$

45 $2\pi[5\sqrt{2} + \ln(1 + \sqrt{2})] \approx 49.97$

47 $2\pi a^2(2 - \sqrt{2}) \approx 3.68a^2$

49 $V(2, 1)$; $F(1, 1)$

51 $V(-4 \pm 3, 0)$; $F(-4 \pm \sqrt{10}, 0)$



53 $(y')^2 = 3x'$