MATH 122: Calculus II

Some Hints and Answers for Assignment 30

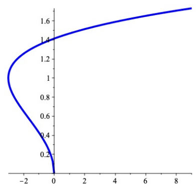
I: Section 8.9: 41, 43

Exercise 41: Maclaurin series for $\cos x$ begins $1 - x^2/2! + 0x^3 + x^4/4!$ so error is of form $\frac{\cos z}{4!}x^4$. $|R_3(x)| \leq \left|\frac{(1)(0.1)^4}{24}\right| = \frac{1}{240000} \approx 4.1 \times 10^{-6} < 0.5 \times 10^{-5}$; accurate to 5 decimal places.

Exercise 43: Maclaurin series for e^x begins $1 + x + x^2/2! + x^3/3!$. $|R_2(x)| \leq \left|\frac{e^{0.1}(0.1)^3}{6}\right| \approx .00018$; 3 decimal places.

II: Section 9.2: 15, 22, 26

Exercise 15: Here $x = 3t^2 - 6t$ has $dx/dt = 6t - 6 = 6(t-1)$ which is 0 when $t = 1$; similarly, $y = \sqrt{t}$ has $dy/dt = \frac{1}{2\sqrt{t}}$ which is 0 at $t = 0$. We have $\frac{dy}{dx} = \frac{1}{12\sqrt{t}(t-1)}$ which is never 0 (so no horizontal tangents) and is undefined (vertical tangents) at $t = 0$ and $t = 1$.. When $t = 0$, the curve is at the point (0,0); at $t = 1$, it is at (-3, 1). To find the second derivative d^2y/dx^2, we first differentiate $\frac{dy}{dx} = \frac{1}{12\sqrt{t}(t-1)}$ with respect to obtain (using the quotient rule) $\frac{1-3t}{24\,t^{3/2}\,(t-1)^2}$ and then divide this expression by $dx/dt = 6(t-1)$ to get $\frac{1-3t}{144\,t^{3/2}\,(t-1)^3}$.



Plot of $x = 3t^2 - 6t,\ y = \sqrt{t},\ t \geq 0$

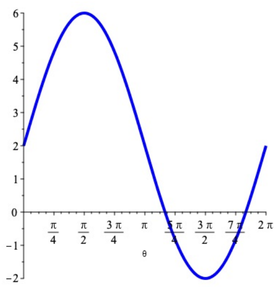
Exercise 22: $(dx/dt)^2 + (dy/dt)^2 = 9 + 9t = 3^2(1+t)$. Length $= 2\left[5^{3/2} - 1\right]$

Exercise 26: Length $= \int_0^{\pi/2} 3\cos t \sin t\, dt = \left|\frac{3}{2}\sin^2 t\right|_0^{\pi/2} = \frac{3}{2}$.

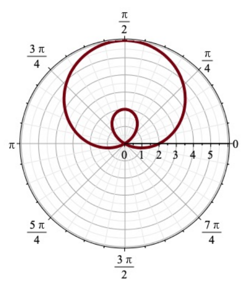
III: Section 9.3: 1, 9, 30

Exercise 1: $r = 5$ is the circle of radius 5 with center at origin.

Exercise 9: $r = 2 + 4\sin\theta$. The curve is a limacon; see the discussion of Examples 2 and 3 in Section 9.3. . Here are helpful pictures:



Cartesian Coordinate plot of $r = 2 + 4\sin\theta$



Polar Coordinate plot of $r = 2 + 4\sin\theta$

Exercise 30: Let $x = r\cos\theta, y = r\sin\theta$. Then $r = 8\sin\theta\sec^2\theta$ which we can also write as $r = 8\tan\theta\sec\theta$