

MATH 122: Calculus II  
*Some Hints and Answers for Assignment 3*  
Page 337: 1, 6, 8, 11, 23, 24, 37, 40

**Don't forget to check endpoints of interval for extrema.**

**Exercise 1:** There is a maximum at  $x = 3$  and a minimum at  $x = 6$ .

**Exercises 6 and 11:**  $f(x) = \frac{1}{x^2+1}$  has  $f'(x) = \frac{-2x}{(x^2+1)^2}$  which is positive for  $x < 0$  and negative for  $x > 0$ . The second derivative is  $f''(x) = \frac{6x^2-2}{(x^2+1)^3}$ ; Since the denominator is positive, the sign of  $f''$  depends on the sign of the numerator. The graph of  $f$  is concave up for  $x < -\sqrt{3}/3$  and  $x > \sqrt{3}/3$  and concave down between these 2 values at which there are points of inflection.

**8:** The function  $f$  is decreasing for  $x < 0$  and increasing for  $x > 0$

**Exercise 23:** The derivative  $f'(x)$  has value 21 when  $x = \frac{-1 \pm \sqrt{61}}{3}$  but only  $x = \frac{-1 + \sqrt{61}}{3}$  is in the interval  $[0, 4]$ .

**24:** 1 hour, 40 minutes  $= (1 + 2/3) = 5/3$  hours. Show that the average velocity of the trip was  $\frac{125}{5/3} = 75$  mi/hr. Then use By the Mean Value Theorem.

**37:** Profit = revenue - cost  $= 18x - (500 + 6x + 0.02x^2)$

(c) Profit is maximized when  $x = 300$ .

(d) Maximum Profit \$1300.

**40:** Let  $p(t)$  be distance moved  $t$  seconds after engaging the brake and  $v(t)$  the velocity at time  $t$ . Let  $S$  be the number of seconds it takes to slow down from 44 ft/sec to 32 ft/sec. Then  $p(0) = 0, p(S) = 114, v(0) = 44, v(S) = 32$  and since deceleration is constant, we have  $v(t) = 44 - \frac{12}{S}t$  because velocity drops 12 ft/sec in  $S$  seconds. Show  $p(t) = 44t - \frac{6}{S}t^2$ . Show  $S = 114/38 = 3$ .

(a) It takes 3 seconds to reduce the speed to 32 ft/sec.

(b) Deceleration  $= 4 \text{ ft/sec}^2$ .

(c) Sled comes to a stop when  $v = 0$ : It takes 11 seconds to reach a full stop.