MATH 122: Calculus II

Some Hints and Answers for Assignment 28

I: Section 8.7: 33, 36, 38

Exercise 33: $f(x) = (1-x^2)^{-1} = \frac{1}{1-x^2}$ is the sum of the geometric series with first term 1 and common ratio x^2 Since $(-1)(-2x)(1-x^2)^{-2} = f'(x)$, wfind its power series representation by term-by-term differentiation of $\sum_{n=0}^{\infty} x^{2n}$

Exercise 36: (a) $\sum_{n=0}^{\infty} p_n = \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} = e^{-\lambda}\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda}\, e^{\lambda} = 1$
(b) Probability(2 or more) = 1 - Probability(less than 2) = 1 - Probability(0) - Probability(1)

Exercise 38: See Example 2, Page 769: Divide power series for $\sin t$ by t and then do term by integration.

II: Section 8.8: 16, 21, 26

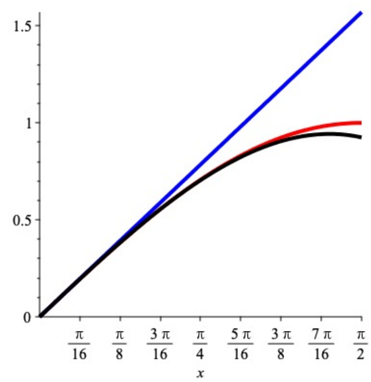
Exercise 16: For $f(x) = \ln(3+x)$, we have $f^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{(3+x)^n}$, so $f^{(n)}(0) = \frac{(-1)^{n-1}(n-1)!}{3^n}$ and hence $a_n = \frac{f^{(n)}(0)}{n!} = \frac{(-1)^{n-1}}{n\,3^n}$

Exercise 21: For $x+1, c = -1$. We have $f(x) = e^{2x}, f'(x) = 2e^{2x}, f''(x)2^2e^{2x}, f^{(3)}(x) = 2^3e^{2x}, ...f^{(n)}(x) = 2^ne^{2x}$ so $f^{(n)}(-1) = \frac{2^n}{e^2}$ and Taylor Series is $\sum \frac{2^n}{e^2\, n!}(x+1)^n$

Exercise 26: Here $f(x) = \arctan x,\ f'(x) = \frac{1}{1+x^2}\ f''(x) = \frac{-2x}{(1+x^2)^2}$ so $f(1) = \arctan 1 = \pi/4,\ f'(1) = 1/2,\ f''(1) = -2/4 = -1/2$ Thus $a_0 = \pi/4,\ a_1 = 1/2,\ a_2 = (-1/2)/2! = -1/4$. The Taylor Series begins $\frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2$.

III: Section 8.9: 1, 7, 13

Exercise 1: (a) We have $P_1(x) = P_2(x) = x, P_3(x) = x - x^3/3!$



(b) Graph of $p_1 = p_2$ in blue, p_3 in black, $\sin x$ in red

(c) $\sin(0.05) \approx P_3(0.05) = 0.04997916667 = \frac{2399}{48000}$. Error $\leq |R_3(0.05)| = \left|\frac{f^{(4)}(z)}{4!}(0.05)^4\right| \leq |(0.05)^4/4!| \approx 2.6\times 10^{-7}$

Exercise 7: $1 - \frac{1}{2!}(x-\frac{\pi}{2})^2 + \frac{\sin z}{4!}(x-\frac{\pi}{2})^4$ for some z between 0 and $\pi/2$.

Exercise 13: For $f(x) = 1/x$, we have $f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}}$ so $f^{(n)}(-2) = \frac{(-1)^n n!}{(-1)^{n+1}2^{n+1}} = \frac{-n!}{2^{n+1}}$ and $a_n = -\frac{1}{2^{n+1}}$. Thus

$$\frac{1}{x} = -\frac{1}{2} - \frac{1}{2^2}(x+2) - \frac{1}{2^3}(x+2)^2 - \frac{1}{2^4}(x+2)^3 - \frac{1}{2^5}(x+2)^4 - \frac{1}{2^6}(x+2)^5 + ...$$

$$R_5(x) = \frac{f^{(6)}(z)(x-c)^6}{6!} = \frac{(-1)^6 6!(x+2)^6}{6!z^7} = \frac{1}{z^7}(x+2)^6, \text{ for some } z \text{ between } x \text{ and -2.}$$