MATH 122: Calculus II
Some Notes on Assignment 27

I: Section 8.6: 43, 45

Exercise 43: Prove that $\sum a_n x^{2n}$ has radius of convergence $\sqrt{r}$ if $\sum a_n x^n$ has radius of convergence r.
Proof: Since $\sum a_n x^n$ is convergent for $|x| < r$ and divergent for $|x| > r$, we know $\sum a_n x^{2n} = \sum a_n (x^2)^n$ converges for $x^2 < r$ or $|x| < \sqrt{r}$ and diverges for $x^2 > r$ or $|x| > \sqrt{r}$. Thus the radius of convergence is $\sqrt{r}$.

Exercise 45: Note that $|(-r)^n| = |r^n|$. Prove the result by contradiction: Suppose that $\sum a_n x^n$ is, in fact, absolutely convergent at $x = r$ and let $x = -r$.

II: Section 8.7: 13, 15, 19

Exercise 13: Begin with $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ so $\arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6}$. Since $\frac{1}{\sqrt{3}} < 1$, we can substitute into the power series representation for $\arctan x$ of Example 5.

Exercise 15: The power series representation $e^x = \sum \frac{x^n}{n!}$ is valid for all real x. Multiply the series for e^{3x} by x.

Exercise 19: If $|x| < 1$, then $|x^2| = |x|^2 < 1$ so power series for $\ln(1 + x)$ is also valid for $\ln(1 + x^2)$.

III: Section 8.8: 1, 6, 9

Exercise 1: $f(x) = e^{3x}$ has $f'(x) = 3e^{3x}$, $f''(x) = 3^2 e^{3x}$, $f^{(3)}(x) = 3^3 e^{3x}$, $f^{(4)}(x) = 3^4 e^{3x}, , .., f^{(n)}(x) = 3^n e^{3x}$ so $f^{(n)}(0) = 3^n e^0 = 3^n$ and so $a_n = \frac{3^n}{n!}$.

Exercise 6: Method 1: $\frac{1}{1-2x}$ is the sum of the geometric series $1 + (2x) + (2x)^3 + (2x)^4 + ... + (2x)^n + ..$
Method 2: Do successive differentiations to show $f^{(n)}(x) = \frac{2^n\, n!}{(1-2x)^{n+1}}$

Exercise 9 $x \sin 3x = x \sum \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!} = x \sum \frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)!} = \sum \frac{(-1)^n 3^{2n+1} x^{2n+2}}{(2n+1)!}$