MATH 122: Calculus II

*Some Hints and Answers for Assignment 26*

## I: Section 8.5: 31, 44, 45

**Exercise 31:** The series $\sum \frac{(-1)^n}{(n-4)^2+5}$ converges absolutely. For large $n$, we have $(n-4)^2+5 \approx (n-4)^2 \approx n^2$ so the series should behave like $\sum 1/n^2$

**Exercise 44:** The series $\sum a_n$ converges. The Ratio Test and Alternating Series Test will be useful.

**Exercise 45:** The claim is false. Consider a converging alternative series $\sum a_n$ and let $b_n = a_n$. You need to choose $a_n$ carefully. Is the claim true if both series are series of positive terms.

## II: Section 8.6: 19, 23, 25

**Exercise 19:** Ratio of Successive Terms approaches $9|x-2|$. The interval of convergence is $[2-1/9, 2+1/9) = [\frac{17}{9}, \frac{19}{9})$.

**Exercise 23:** In the limit of the ratio of successive terms we have one factor approaching 1, one approaching $|x-3|$, one approaching $e$ and one of the form $n+1$ so the ratio has limit $\infty$ for all $x$ except 3 where limit is 0. Thus the series converges only at $x = 3$.

**Exercise 25:** Here the Ratio of Successive Terms reduces to

$$\left(\frac{\ln(n+1)}{\ln n}\right)\frac{1}{e}|x-e|$$

Use l'Hôpital's Rule to find the limit. The interval of convergence is $(0, 2e)$.

## III: Section 8.7: 1, 5

**Exercise 1:** $f(x) = \frac{1}{1-3x}$ is the sum of the geometric series with first term 1 and common ratio $3x$. By Theorem 8.40, $f'(x) = \sum_{n=0}^{\infty} n3^n x^{n-1}$ and $\int f(x) = \sum_{n=0}^{\infty} 3^n \frac{x^{n+1}}{n+1}$

**Exercise 5** $\frac{x^2}{1-x^2} = x^2 \frac{1}{1-x^2}$ which is $x^2$ times the geometric series with first term 1 and common ratio $x^2$.