MATH 122: Calculus II: *Some Hints and Answers for Assignment 24*

I: Section 8.3: 43, 49, 57

Exercise 43: Start with $n + 3^n > 3^n$ so $\frac{1}{n+3^n} < \frac{1}{3^n}$ and thus $\frac{n^2+2^n}{n+3^n} < \frac{n^2+2^n}{3^n}$. Note also that $n^2 < 2^n$ for $n > 3$. Our converges converges by comparison to a geometric series with ratio $\frac{2}{3}$.

Exercise 49: From the results of the proof of (8.23) with $f(x) = \frac{1}{x+1}$ and then $f(x) = 1/x$, we have

$$(i) \qquad \sum_{k=2}^{n} \frac{1}{k+1} \le \int_1^n \frac{1}{x+1}\,dx \le \sum_{k=1}^{n-1} \frac{1}{k+1} \text{ which implies}$$

$$\frac{1}{3} + \frac{1}{4} + ... + \frac{1}{n+1} \le \ln(n+1) - \ln 2 \le \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} \text{ implying}$$

$$(\text{since } \ln 2 < 1) \ln(n+1) < 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$$

$$(ii) \qquad \sum_{k=2}^{n} \frac{1}{k} \le \int_1^n \frac{1}{x}\,dx \le \sum_{k=1}^{n-1} \frac{1}{k} 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} \le 1 + \ln n \text{ which implies}$$

$$\frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} \le \ln n \le 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n-1} \text{ implying}$$

$$1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} \le 1 + \ln n$$

$$\text{By (i) and (ii), } \ln(n+1) \le 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} \le 1 + \ln n \quad (n > 1)$$

Exercise 57: If $\sum a_n$ converges, then $a_n \to 0$ so $\frac{1}{a_n}$ diverges. Apply nth-term Test.

II: Section 8.4: 13, 17, 21

Exercise 13: $\lim_{n\to\infty} \sqrt[n]{a_n} = \frac{2}{1}$ You may need l'Hopital's Rule to find that limit is indeed 2.

Exercise 17: $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n} = \frac{1}{2}$. Series Converges

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Exercise 21: Ratio Test

$$\frac{a_{n+1}}{a_n} = \left(\frac{99^{n+1}((n+1)^5+2)}{(n+1)^2 10^{2(n+1}}\right)\left(\frac{10^{2n}n^2}{99^n(n^2+1}\right) \to = \frac{99}{100} < 1 \text{ so series converges.}$$

III: Section 8.5: 1, 6,11 Alternating Series Test applies for all three series.