MATH 122: Calculus II

Some Hints and Answers for Assignment 23

I: Section 8.2: 43, 57, 61

Exercise 43: Difference of two convergent geometric series? $\sum(2^{-n} - 3^{-n}) = 1 - \frac{1}{7} = \frac{6}{7}$.

Exercise 57: Consider $\sum(a_n - a_n)$ where $\sum a_n$ diverges.

Exercise 61: (a) Use the fact that the amount of the drug in bloodstream immediately after the kth dosage is $Q+$ amount left in the blood stream after the $(k-1)$st dosage times e^{-cT}; thatis, $A(k) = Q + A(k-1)e^{-cT}$.
(b) $A(k) < \frac{Q}{1=-e^{-cT}}$
(c) We want $\frac{Q}{1-e^{-cT}} < M$ so $T > -\frac{1}{c}\ln\frac{M-Q}{M}$.

II: Section 8.3: 22, 29, 36

Exercise 22: Comparison Test: $3+\sqrt{n} < \sqrt{n}+\sqrt{n}(n > 9)$ so $\frac{1}{3+\sqrt{n}} > \frac{1}{2\sqrt{n}}$ and $\frac{1}{2}\sum\frac{1}{\sqrt{n}}$ which diverges by p-series test with $p = 1/2$. We can also use Limit Comparison Test with $a_n = \frac{1}{3+\sqrt{n}}$ and $b_n = \frac{1}{\sqrt{n}}$ where $\lim_{n\to\infty}(a_n/b_n) = 1$

Exercise 29: For large values of n, $2n+n^2 \approx n^2$ and $n^3+1 \approx n^3$ so $\frac{2n+n^2}{n^3+1}$ should behave like $\frac{n^2}{n^3} = \frac{1}{n}$. You could also use a direct comparison test showing $a_n > b_n$ for all n.

Exercise 36: The series should behave like the convergent series $\sum\frac{1}{n^2}$
Using Limit Comparison with $a_n = \frac{n+\ln n}{n^3+n+1}$ and $b_n = \frac{1}{n^2}$.

III: Section 8.4: 1,5, 9

Exercise 1: Use Ratio Test: Ratio has limit 1/2.

Exercise 5: Ratio has limit 0.

Exercise 9: Ratio Test:

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{e} \to \infty \text{ as } n \to \infty$$