MATH 122: Calculus II

Some Hints and Answers for Assignment 22

I: Section 8.1: 40, 41, 43

Exercise 40: $\{(-1)^n \frac{n^2}{1+n^2}\}$ Sequence diverges. Start with $\frac{n^2}{1+n^2} = \frac{1}{\frac{1}{n^2}+1}$.

Exercise 41: $\{\sqrt{n+1} - \sqrt{n}\}$ is n indeterminate $\infty - \infty$ form. Rationalize the expression. The sequence converges to 0.

Exercise 43: (a) Population on A at year $n+1$ is made up of the 90 percent who were on the island at year n and remained plus 5 percent of the population on island C who moved to island A. Thus $A_{n+1} = .9A_n + .05C_n$. The equations $B_{n+1} = .1A_n + .8B_n$ and $C_{n+1}.95C_n + .2B_n$ follow from similar reasoning
(b) In the limit, as $n \to \infty$, we have

$$\lim_{n\to\infty} A_{n+1} = A = \lim_{n\to\infty} A_n, \ \lim_{n\to\infty} B_{n+1} = B = \lim_{n\to\infty} B_n, \ \lim_{n\to\infty} C_{n+1} = C = \lim_{n\to\infty} C_n$$

. Thus we have

$$A = .9A + .05C \text{ implying } .1A = .05C \text{ so } C = (.1/.05)A = 2A$$
$$B = .1A + .8B \text{ implying } .2B = .1A \text{ so } B = (.1/.2)A = A/2, \ C = .95C + .2B$$

II: Section 8.2: 20, 27, 33

Exercise 20: This is a geometric series with first term = 3 and common ratio $\frac{x-1}{3}$. For $-2 < x < 4$, the series converges to $\frac{9}{4-x}$.

Exercise 27: Here $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$ so we have a telescoping series.

Exercise 33: Series diverges as it fails the nth term test.

III: Section 8.3: 1, 8, 15

Exercise 1: Show the Integral Test applies. The series converges.

Exercise 8: Show that the Integral Test applies. The improper integral $\int_2^\infty \frac{1}{x(\ln x)^2}\, dx$ has a finite limit.

Exercise 15: Try comparing this series with a geometric series with ratio 1/3.