MATH 122: Calculus II

Some Hints and Answers for Assignment 19

I: Section 7.7: 47, 67, 84a

Exercise 47: $\int_0^8 \frac{1}{x^{\frac{1}{3}}} dx$ is improper because the integrand is not defined at $x = 0$ and becomes unbounded as x approaches 0 from the right. The integral converges to 6.

Exercise 67: $\frac{x-2}{x^2-5x+4} = \frac{x-2}{(x-1)(x-4)}$ is undefined at $x = 4$ and grows unboundedly as x approaches 4 from the left so the integral is improper. Use a partial fraction decomposition. The original given integral diverges.

Exercise 84a: Make use of two earlier results: (1) $\int \ln x \, dx = x \ln x - x + C$ from Example 3 on Page 632 and (2) $\lim_{x \to 0^+} x \ln x = 0$ from Exercise 49 of Section 6.9, done in Assignment 14.
First solve $N = N_0 e^{-at}$ for $t = -\frac{1}{a} \ln(N/N_0) = -\frac{1}{a}[\ln N - \ln N_0]$.

II: Section 9.1: 22, 27a, 29, 49, 52ab

Exercise 22: $y = 8(\frac{x^3}{8}) = x^3$

Exercise 27a: $x = \cos t, y = \sin t, 0 \leq t \leq \pi$ traces out the top half of the unit circle (center at origin, radius 1) counterclockwise from (1,0) to (-1,0).

Exercise 29 Show that $\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$, the standard definition of such an ellipse.

Exercise 49: Let $x(t), y(t)$ be horizontal and vertical coordinates of rock at time t where initial position is (0,3). Show $x(t) = 90t$ and $y(t) = -16t^2 + 47t + 3 = -(16t + 1)(t - 3)$. Show that the rock reaches the position of the fence when $= t = 1/10$ at which time its height above the ground 7.54 ft.

Exercise 52ab: (a) Show vertical position of ball as $y(t) = -16t^2 + 24t = 8t(3 - 2t)$ and horizontal position $x(t) = 24t$. The ball hits the ground at time $t = 3/2$ where it is $x(3/2) = 36\sqrt{3} \approx 62.35$ feet from Annie. Sasha's distance from Annie is given by $90 - 20t$

(b) Ball is at height $5.5 = 11/2$ feet at time T when $y(T) = 11/2$; this happens when $T = \frac{6+\sqrt{14}}{8} \approx 1.2177$ seconds.

III: Section 8.1: 1, 7, 13

Exercise 1: $a_n = \frac{1}{3+2/n} \to \frac{1}{3}$

Exercise 7: Divide numerator and denominator by n^3.

Exercise 13: $a_n = 1 + (0.1)^n$ has $a_1 = 1.0, a_2 = 1.01, a_3 = 1.001, a_4 = 1.0001$. $a_n \to 1$.