MATH 122: Calculus II

Hints and Answers for Assignment 18

I: Section 7.4: 34, 37

Exercise 34: $\frac{1}{u(a+bu)} = \frac{A}{u} + \frac{B}{a+bu} = \frac{Aa+(Ab+B)u}{u(a+bu}$.. We need $Aa = 1$ and $Ab + B = 0$ so $A = 1/a, B = -b/a$. The integral $= \frac{1}{a} \ln |\frac{u}{a+bu}| + C$

Exercise 37: Note that $f(x) = \frac{x}{x^2-2x-3}$ is negative throughout the interval [0,2] so the area is $-\int_0^2 f(x)\,dx$. Now $f(x) = \frac{A}{x+1} + \frac{B}{x-3}$ with $A = 1/4, B = 3/4$. The area is $\frac{1}{2}\ln 3 \approx 0.549$.

II: Section 7.7: 21, 30, 34

Exercise 21: $\int_1^\infty \frac{1}{x^4}\,dx$ converges and $1 + x^4 > x^4$

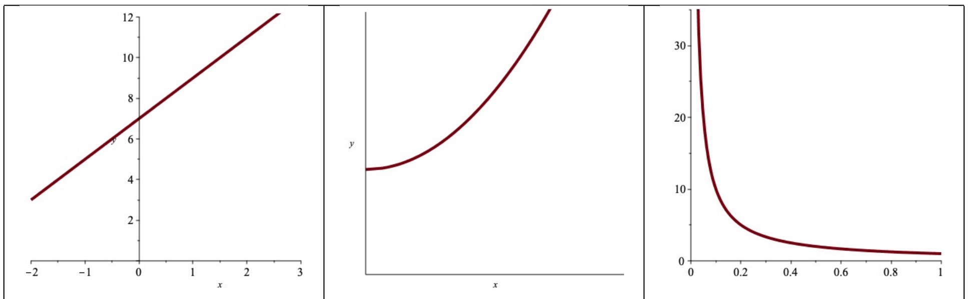
$$\int_1^\infty \frac{1}{x^4}\,dx == \frac{1}{3}$$

Exercise 30: (a) $\int_0^\infty R(t)\,dt = \int_0^\infty mke^{-kt}\,dt$ is total mass of fuel burned over all future time.
(b) The amount burned in the first T seconds is $\int_0^T mke^{-kt}\,dt = m(1-e^{-kT})$. The spacecraft will never run out of fuel.

Exercise 34: If k is a positive constant, then $\int 12000e^{-kt}\,dt = -12000\frac{1}{ke^{kt}}$ so $\int_T^b 12000e^{-kt}\,dt$ has limit $A = 12000\frac{1}{ke^{kT}}$ as $b \to \infty$. For $T = 20$, we have $A = 12000\frac{1}{ke^{20k}}$.
(a) Here $k = .08$ so $A = \frac{12000}{.08e^{.16}} \approx \$30,284$
(b) Integrate $12000e^{.04t}e^{-.08t} = 12000e^{-.04t}$ so $k = .04$ and $A \approx \$134,799$.

III: Section 9.1: 1, 8, 15

Exercise 1: $x = t-2, y = 2t+3, 0 \le t \le 5$. $x = t-2$ implies $t = x+2$. Then $y = 2(x+2)+3 = 2x+7$ Graph is straight line from (-2, 3) to (3, 13).



Exercise 1 Exercise 8 Exercise 15

Exercise 8: $x = \sqrt{t}, y = 3t+4, t \ge 0$: We have $x^2 = t$ so $y = 3x^2 + 4$. Graph is right hand side of parabola.

Exercise 15: $y = \csc t = \frac{1}{\sin t} = \frac{1}{x}$. Graph is hyperbola: (x, y) varies asymptotically from positive vertical axis to (1,1)