MATH 122: Calculus II

Hints and Answers for Assignment 16

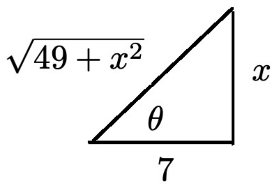
I: Section 7.2: 25, 29

Exercise 25: Note that $\csc^4 x \cot^4 x = \cot^4 x \csc^2 x \csc^2 x = \cot^4 x(1{+}\cot^2 x)\csc^2 x = \cot^4 x \csc^2 x \cot^6 x \csc^2 x$ and use the fact that the derivative of $\cot x$ is $-csc^2 x$. Let $u = \cot x$, then $du = -\csc^2 x\, du$.

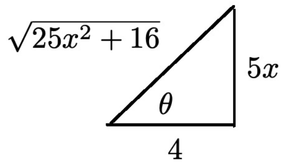
Exercise 29: Since $\sec^2 x$ is the derivative of $\tan x$, a reasonable substitution to try is $u = 1 + \tan x$.

II: Section 7.3: 14, 18, 22

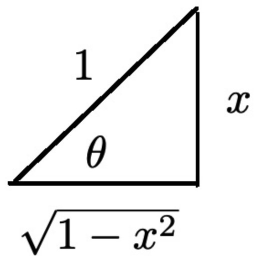
Exercise 14: $\int \frac{1}{49+x^2}\, dx$. Set up right triangle with x as as opposite side and 7 as adjacent side, making hypotenuse $\sqrt{49+x^2}$; see picture below. Then our substitutions are $\tan\theta = x/7$ so $x = 7\tan\theta$ and $dx = 7\sec^2 x\theta\, d\theta$ with $49+x^2 = 49\sec^2\theta$. Then $\int \frac{1}{49+x^2}\, dx = \int \frac{7\sec^2\theta}{49\sec^2\theta}\, d\theta = \frac{1}{7}\int 1\, d\theta = \frac{1}{7}\arctan(x/7)+C$



Exercise 14: $\int \frac{1}{49+x^2}\, dx$



Exercise 18: $\int \frac{1}{x\sqrt{25x^2+16}}\, dx$



Exercise 22: $\int \frac{3x-5}{\sqrt{1-x^2}}\, dx$

Exercise 18: c Set up right triangle with $5x$ as opposite side and 4 as adjacent side. Then hypotenuse is $\sqrt{25x^2+16}$; see picture above. Simplest ratio involving x is $\tan\theta = 5x/4$ so use $x = \frac{4}{5}\tan\theta$

Exercise 22: $\int \frac{3x-5}{\sqrt{1-x^2}}\, dx$ Set up right triangle with x as opposite side and 1 as hypotenuse so adjacent side is $\sqrt{1-x^2}$.

III: Section 7.4: 2, 3, 5

Exercise 2: $\frac{x+34}{(x-6)(x+2)} = \frac{A}{x-6} + \frac{B}{x+2}$ $\int \frac{x+34}{(x-6)(x+2)}\, dx = \int \frac{5}{x-6} + \frac{-4}{x+2} dx = 5\ln|x-6| - 4\ln|x+2| + K$

Exercise 3: $\frac{37-11x}{(x+1)(x-2)(x-3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-3} = \frac{A(x-2)(x-3)+B(x+1)(x-3)+C(x+1)(x-2)}{(x+1)(x-2)(x-3)}$
Thus (*)$37 - 11x = A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)$
Step 1: Set x = -1 in (*): 37 - 11(-1) =A(-3)(-4) + 0 + 0 so 48 = 12A which gives $A = 4$
Step 2: Set x = 2 in(*): 15 = B(3)(-1) so B = -5
Step 3: Set x = 3 in (*): 4 = $C(4)(1)$ so $C = 1$

Exercise 5: $\frac{6x-11}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$
$\int \frac{6x-11}{(x-1)^2}\, dx = \int \frac{6}{x-1} - \frac{5}{(x-1)^2}\, dx = 6\ln|x-1| + \frac{5}{x-1} + K$