MATH 122: Calculus II

Hints and Answers for Assignment 14

I: Section 6.9: 49, 51, 60, 82

We use $=^{LH}$ to indicate that the equality follows from applying l'Hôpital's Rule.

Exercise 49: $x \ln x$ is of the indeterminate form $0 \cdot \infty$ as $x \to 0^+$. Rewrite expression as $\frac{\ln x}{1/x}$. The limit is 0.

Exercise 51: $(x^2-1)e^{-x^2}$ is of the indeterminate form $\infty \cdot 0$ as $x \to \infty$. Rewrite the expression as $\frac{x^2-1}{e^{xx^2}}$ which then is an $\frac{\infty}{\infty}$ form and we can use l'Hôpital's Rule. The limit is also 0.

Exercise 60: $y = x^{1/x}$ is of indeterminate form ∞^0 as $x \to \infty$. Note that $\ln y = \frac{1}{x} \ln x = \frac{\ln x}{x}$ is an 0/0 form. We can use l'Hôpital's Rule on $\ln y$: $\lim_{x\to\infty} \ln y = 0$. Finally, $y \to 1$.

Exercise 82: $y = (1+\frac{r}{m})^{mt}$ is an indeterminate 1^∞ form as $m \to \infty$. Work with $\ln y = mt \ln\left(1+\frac{r}{m}\right) = \frac{\ln\left(1+\frac{r}{m}\right)}{1/mt}$ which is an 0/0 form

II: Section 7.1: 19, 24, 31

You can check your work by differentiating your answer.

Exercise 19: Let $u = \ln \cos x$ and $dv = \sin x\, dx$

Exercise 24: Let $I = \int \sin \ln x\, dx$ and $J = \int \cos \ln x\, dx$. Use integration by parts with $u = \sin \ln x, dv = dx$ on I and $U = \cos \ln x, dV = dx$ on J.

Exercise 31: Recall Example 3 on Page 632 where we used integration by parts to show that $\int x \ \ln x\, dx = x \ln x - x$ and write $\int (\ln x)^2\, dx$ as $\int (\ln x)(\ln x)\, dx$.

III: Section 7.2: 1, 5, 9

Exercise 1: $\cos^3 x = (\cos^2 x)(\cos x) = (1-\sin^2 x)(\cos x)$ so let $u = \sin x$.

Exercise 5: $\sin^3 x \cos^2 x = \sin^2 x \cos^2 x \sin x = (1-\cos^2 x)\cos^2 x \sin x = (cos^2 x - \cos^4 x)\sin x$. Let $u = \cos x$ so $du = -\sin x\, dx$

Exercise 9: $\tan^3 x \sec^4 x = \tan^3 x \sec^2 x \sec^2 x = \tan^3 x(1+\tan^2 x)\sec^2 x = (\tan^3 x + \tan^5 x)\sec^2 x$. Let $u = \tan x$ so $du = \sec^2 x\, dx$.
Then $\int \tan^3 x \sec^4 x\, dx = \int (u^3+u^5)\, du = \frac{u^4}{4} + \frac{u^6}{6} + C = \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C$.