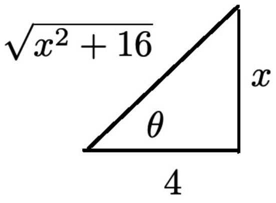
MATH 122: Calculus II

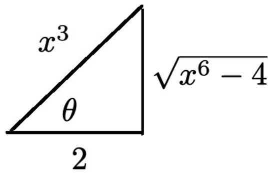
Hints and Answers for Assignment 13

I: Section 6.7: 51, 60, 69

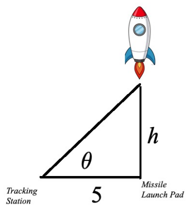
Exercise 51: Let $\tan\theta = \frac{x}{4}$. $\int_0^4 \frac{1}{x^2+16}\,dx = \frac{\pi}{16}$



Exercise 51



Exercise 60



Exercise 69

Exercise 60: Consider a right triangle with hypotenuse x^3 and horizontal side 2; then the vertical side is $\sqrt{x^6-4}$. The simplest ratio in the triangle involving x is $\frac{x^3}{2}$ which is $\sec\theta$. See picture above. $\int \frac{1}{x\sqrt{x^6-4}}\,dx = \frac{1}{6}\,\text{arcsec}\left(\frac{x^3}{2}\right) + C$

Exercise 69: (See Figure above): Let $h(t)$ be height of missile t seconds after firing and θ the angle of elevation. We are looking for $h'(t)$ at the instant $\theta = 30°= \pi/6$ radians. Switch to radian measure. One relation between h and θ that is true at every second is $\tan\theta = h/5$ so $h(t) = 5\tan\theta(t)$. At the given instant, we have $h' = \frac{2\pi}{27} mi/sec$.

II: Section 6.9: 28, 36, 42

Exercise 28: Limit is 0 (l'Hôpital's Rule does **not** apply!)

Exercise 36: Let $u = \frac{1}{x}$. Then $\frac{e^{-1/x}}{x} = \frac{e^{-u}}{1/u} = \frac{u}{e^u}$. Limit is 0.

Exercise 42: (a) K; l'Hôpital's Rule does not apply here).

(b) $y(t)$ is a $\frac{\infty}{\infty}$ form as $K \to \infty$. Limit is $y(0)e^{rt}$ Then consider 2 cases: K is unbounded and K is bounded.

III: Section 7.1: 1, 7, 13

Integration By Parts Formula: $\int u\,dv = uv - \int v\,du$

Exercise 1: Let $u = x$ and $dv = e^{-x}dx$. $\int xe^{-x}\,dx = - - e^{-x}(x+1) + C$.

Exercise 7: Let $u = x$ and $dv = \sec x\tan x\,dx$. Then $\int x\sec x\tan x\,dx = x\sec x - \ln(|\sec x + \tan x|) + C$.

Exercise 13: $\int \sqrt{x}\,\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$.