MATH 122: Calculus II

Some Hints and Answers for Assignment 12

I: Section 6.6: 21, 22

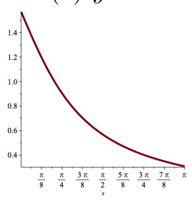
Exercise 21: $I(x) = I_0 e^{-f(x)}$ has $I'(x) = I_0 e^{-f(x)} \times (-f(x))' = -I(x)(f'(x))$ but $f(x) = k \int_0^x p(h)\, dh$ so the Fundamental Theorem of Calculus yields $f'(x) = kp(x)$ and thus $I'(x) = -kp(x)I(x)$

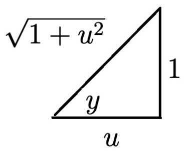
Exercise 22: $f(n) = 3 + 20\left(1 - e^{-0.1n}\right)$
(a) $f(5) \approx 10.87$; $f(9) \approx 14.87$; $f(24) \approx 21.19$; $f(30) \approx 22$
(b) $f'(n)$ is positive for all n while $f''(n)$ is negative so graph of f is increasing and concave down.
(c) $f(n) = 23 - e^{-0.1n}$;exponential term goes to 0 as n gets large.

II: Section 6.7: 27, 30, 41

Exercise 27: (a) $y = \operatorname{arccot} x$ means $x = \cot y$ for any real number x and $0 < y < \pi$.

(b)

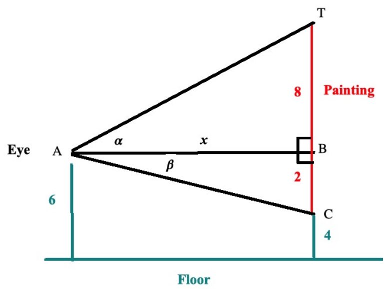




Graph of $\operatorname{arccot} x$ on $[0, \pi]$ Visualizing $\cot y = u$

(c) If $y = \operatorname{arccot} u$, then $\cot y = u$. Taking derivatives with respect to x we have $(-\csc^2 y)\, y' = u'$ so $y' = (-\sin^2 y)(u'(x)) = -\frac{1}{1+u^2} u'(x) = (-\arctan y)'$

Exercise 30: Draw lines from the critic's eyes to the top of the painting and to the bottom of the painting. See picture below. Let $x$be the distance between the critic and the painting. We let α be the angle of elevation to the painting's top and β the angle of depression to the painting's bottom.



(a) We have a right triangle ATB with $\tan\alpha = 8/x$ and another right triangle ABC with $\tan\beta = 2/x$. Note also that $\theta = \alpha + \beta = \arctan(8/x) + \arctan(2/x)$.
(b) Use $\tan\theta = \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - (\tan\alpha)(\tan\beta)} = \ldots = \frac{10x}{x^2-16}$.
Note that to get a positive angle, we need $x > 4$
(c) Since the tangent of 45°=1, we need to solve $\frac{10x}{x^2-16}. = 1$; that is $x^2 - 16 = 10x$ or $x^2 - 10x - 16 = 0$. Apply the quadratic formula to obtain $x = 5 \pm \sqrt{41}$.Since $x > 0$, we have $x = 5 + \sqrt{41} \approx 11.4$ feet.

Exercise 41: $f'(x) = -\sin(\frac{1}{x})\,(-\frac{1}{x^2}) + \sec x \tan x - \frac{1}{\sqrt{1-x^2}} = \sin(\frac{1}{x})\,(\frac{1}{x^2}) + \sec x \tan x - \frac{1}{\sqrt{1-x^2}}$

III: Section 6.9: 1, 10, 19

These 3 problems can be done with or without l'Hôpital"s Rule

Exercise 1: $\frac{1}{2}$

Exercise 10: 0

Exercise 19: By 2 applications of l'Hôpital, limit is $\frac{2}{5}$