MATH 122: Calculus II
Hints and Answers for Assignment 11

I: Section 6.5: 45, 48, 53

Exercise 45 : The area is $\int_0^1 2^x - (1-x)\,dx$

Exercise 48: The amount accumulated by time t is $A(T) = \int_0^T 5(.95^t)dt = \left[\frac{5}{\ln .95}.95^t\right]_0^T = \frac{5}{\ln .95}\left[.95^T - 1\right]$. We want to choose T so that this amount is 50: $50 = \frac{5}{\ln .95}\left[.95^T - 1\right]$. $T = \frac{\ln(1+10\ln .95)}{\ln .95} \approx 14.02$ minutes.

Exercise 53: (a) $R(x) = a\log\left(\frac{x}{x_0}\right)$ so $R(x_0) = a\log\left(\frac{x}{x_0}\right) = a\log 1 = a(0) = 0$

(b) We can also write $R(x)$ as $R = a\log\left(\frac{x}{x_0}\right) = a\log x - a\log x_o$ where second term is constant. Then $S(x) = dR/dx = \frac{a}{\ln 10}\frac{1}{x} = \frac{k}{x}$ where $k = \frac{a}{\ln 10}$ is constant; Thus S is inversely proportional to x.
Finally, note $S(2x) = \frac{k}{2x} = \frac{1}{2}\ \frac{k}{x} = \frac{1}{2}S(x)$ so $S(x) = 2S(2x)$.

II: Section 6.6: 10, 15, 19

Exercise 10: Let $N(t)$ be the number of ticks per minute t days after it was 2000. . Then $N(t) = 2000e^{rt}$. Half life is at time T where $N(T) = 1000$ Solve T to get $T/10\ln(3/4) = \ln(1/2)$ so $T = \frac{10\ln(1/2)}{\ln(3/4)} \approx 24.09$ days

Exercise 15: Let $U(t)$ be number of units of the drug t hours after the operation. Then $U(t) = U_0e^{rt}$.

$$U_0 = 600e^{-(\frac{3}{4})(-\frac{\ln 2}{4})} = 600e^{\frac{3}{16}\ln 2} = 600e^{\ln(2^{3/16}} = 600\left(2^{\frac{3}{16}}\right) \approx 683.27$$

Exercise 19: Let $q(t)$ be amount of ^{14}C present at t years. Then $q(t) = q_0e^{at}$. $T = \frac{-(\ln 5)5700}{-\ln 2} \approx 13,235$ years.

III: Section 6.7: 1, 9, 18

Exercise 1: (a) $-\pi/4$; (b) $2\pi/3$; (c) $-\pi/3$

Exercise 9: (a) $\sqrt{3}/2$ (b) 0
(c) undefined..

Exercise 18: To find $\sec(\arcsin x/3)$, draw a right triangle with angle θ that has $\sin\theta = x/3$. Let opposite side by x and hypotenuse 3. Find adjacent third side by Pythagorean Theorem.