MATH 122: Calculus II

Hints and Answers for Assignment 10

I: Section 6.4: 45, 48, 52

Exercise 45 : Let $u = x^2 + 4$. $\int_0^3 f(x)\, dx = \frac{c}{2} \ln\left(\frac{13}{4}\right)$

Exercise 48 (a) Value $= v(t) = 500e^{.07t}$ which is 1000 when $t = \frac{\ln 2}{.07}$
(b) We want T so that $v'(T) = 50$; so $T = (\frac{100}{7}) \ln(10/7) \approx 5.1$ years.

Exercise 52: Let $C(t)$ be the amount consumed t years in the future. Then $C'(t) = R(t) = 6.5e^{.02t} = 6.5e^{t/50}$ and hence C is the integral of R. We seek the value T where $C(T) = \int_0^T 6.5e^{t/50}\, dt = 50$. T $T = 50 \ln \frac{15}{13} \approx 7.16$ years.

II: Section 6.5: 24, 31, 38

Exercise 24: (a) π^π is a constant (b) and (c); Power rule (d) $(\pi^x)' = \ln \pi(\pi^x)$. (e) Let $y = x^{2x}$ so $\ln y = 2x \ln x$. We find $(x^{2x})' = x^{2x}\,(2 \ln x + 2)$

Exercise 31: Let $u = -2x$ so $dx = -\frac{1}{2}du$. Then $\int 5^{-2x}\, dx = -\frac{1}{2} \int 5^u\, du = (-\frac{1}{2})\frac{1}{\ln 5}5^{-2x} + C$

Exercise 38: Let $u = 3^x + 4$ so $du = (\ln 3)3^x\, dxx$. Then $\int \frac{3^x}{\sqrt{3^x+4}}\, dx = \frac{1}{\ln 3}2\sqrt{3^x + 4} + C$

III: Section 6.6: 1, 5, 7

Exercise 1: If $q(t)$ is the number of bacteria at time t, then $q(t) = 5000e^{ct}$ for some constant c. Since $q(10) = 15000$, we have $5000e^{10c} = 15000$ so $e^{10c} = 3$ which gives $10c = \ln 3$ and $c = \frac{\ln 3}{10}$ and $q(t) = 5000e^{\frac{\ln 3}{10}t}$. The number of bacteria after 20 hours is $q(20) = 5000e^{\frac{\ln 3}{10}20} = 5000e^{2\ln 3} = 5000e^{\ln 9} = 9(5000) = 45,000$.
The number of bacteria reaches 50,000 at a time T such that $50,000 = q(T) = 5000e^{\frac{\ln 3}{10}T}$ so $e^{\frac{\ln 3}{10}T} = \frac{50000}{5000} = 10$. Taking logarithms, we have $\frac{\ln 3}{10}T = \ln 10$ so $T = 10\frac{\ln 10}{\ln 3} \approx 20.96$ hours.

Exercise 5: The population P at time t is $P(t) = P_0e^{rt}$ where $P_0 = 5.5$ and $r = .02 = 1/50$. We need to find T so $P(T) = 40$. $T = 50 \ln(40/5.5) \approx 99.2$ years after January 1, 1993

Exercise 7: (See class notes). $T(t) = 70 - 30e^{-(t/5)\ln 3}$. Temperature reaches 65 when

$$t = 5\,\frac{\ln(1/6)}{\ln(1/3)} \approx 8.15 \text{ minutes}$$