MATH 122: Calculus II

Some Hints and Answers for Assignment 29

I: Section 8.8: 30, 48

Exercise 30: Replace x with $-xx$ in the power series representation of the exponential function. Error is no larger than third term,1/2.

Exercise 48: Use 8.48(e). Using these 5 nonzero terms, we have $\arctan\frac{1}{2} \approx \frac{74783}{161280} \approx .463684$, $\arctan\frac{1}{3} \approx \frac{1994903}{6200145} \approx .321751$ and $\pi = 4(\arctan\frac{1}{2} + \arctan\frac{1}{3} \approx 3.14174$.

II: Section 8.9; 20, 26, 28

Exercise 20: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{\sin z}{8!}x^8$

Exercise 26: The Taylor Series ($n = 3$) with remainder $R_3(x)$

$$2 - \frac{1}{4}x - \frac{1/32}{2!}x^2 - \frac{3/256}{3!}x^3 + \left(\frac{-15}{16(4-z)^{7/2}}\right)\frac{1}{4!}x^4 = 2 - \frac{1}{4}x - \frac{1}{64}x^2 - \frac{1}{512}x^3 + \frac{-15}{128(4-z)^{7/2}}x^4$$

Exercise 28: Determine the first several derivatives of $f(x) = e^{-x^2}$ The Taylor Series ($n = 3$) with remainder $R_3(x)$:

$$1 + \frac{0}{1!}x - \frac{2}{2!}x^2 + \frac{0}{3!}x^3 + \frac{f^{(4)}(z)}{4!}x^4 = 1 - x^2 + \frac{4e^{-z^2}(3 - 12z + 4z^2)}{4 \cdot 3!}x^4$$

III: Section 9.2: 1,7, 10

Exercise 1: The slope of tangent line is $dy/dt = \frac{dx/dt}{dy/dt} = \frac{2t}{2t} = 1$. Slope of normal line -1.

Exercise 7: At $t = 1$, slope of tangent line is $-\frac{3}{2}\tan 1$; slope of normal line is thus $\frac{2}{3}\cot 1$.

Exercise 10: The slope of the tangent line is $\frac{10t}{2t+1}$ which equals 4 when $t = 2$. The point is (6,17).