MATH 122A Calculus II
Sample Examination 3

1. Define what it means for a sequence to **bounded**

2. .(a) Determine whether the sequence {an} where an = $1/5^n$ is increasing, decreasing, or not monotonic. Is the sequence bounded? Does it converge?
(b) Give an example of a sequence which bounded but does not converge.

3. For each of the two series below, determine if it converges or diverges. If it converges, find the sum:
(a) 1 + 0.4+0.16 + .0064 + … (b) $\sum_{n-1}^{\infty} \frac{(-6)^{n-1}}{5^n}$

4. Determine whether the series is convergent or divergent:
$$\sum_{n=1}^{\infty} \frac{7n - n^{1/3}}{n^5}$$

5. Use the integral test to determine if the following series converges or diverges
$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$

6. Test the series for convergence
$$\sum_{n=1}^{\infty} (-1)^n \frac{2n}{4n^2 + 1}$$

7. Test for absolute convergence
$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$$

8. Find the interval of convergence of the power series
$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$$

9. (a) Either give an example of an infinite series that sums to 10^{2023} or show that no series can add up to that large a number.
(b) Suppose $\{a_n\}$ is a sequence of positive numbers such that $\sum a_n$ converges. Provide a careful argument that $\sum a_n^2$ must also converge.

10. Determine which of the following improper integrals converge and which diverge:
(a) $\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$ (b) $\int_0^{1} \frac{1}{\sqrt{x}}\, dx$ (c) $\int_0^{\infty} \frac{1}{1+x^2}\, dx$ (d) $\int_2^{4} \frac{1}{x-4} dx$

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