MATH 122A Calculus II
Sample Examination 2

1. Find the derivative with respect to x of each of the following:
    (a) 2^{2-x^2} (b) $log_2(x^3 - 5x^2 + 3)$ (c) arcsin x (d) arctan $(3x - 5)$

2. Determine each of following integrals:
    (a) $\int_{-2}^{1} 7^x \, dx$ (b) $\int \ln x \, dx$ (c) $\int_0^4 \frac{1}{x^2+16} \, dx$ (d) $\int x^2 \cos x \, dx$

    (e) $\int_0^{\frac{\sqrt{2}}{2}} \frac{x}{\sqrt{1-x^4}} \, dx$ (f) $\int \frac{1}{x\sqrt{x^6-4}} \, dx$ (g) $\int \sin^3 x \, dx$ (h) $\int \frac{2x-7}{(x-3)(x-4)} \, dx$

3. Find each of these limits :
    $\lim\limits_{x\to0} \frac{5x}{\tan x}$ (b) $\lim\limits_{x\to\infty} \frac{x-\cos x}{x}$ (c) $\lim\limits_{x\to\frac{\pi}{2}}(1 + \cos x)^{\tan x}$

4. Consider 2 new functions $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$. These are called the *hyperbolic sine* and *hyperbolic cosine* functions, respectively. They have many applications in science and engineering.
    (a) Show that $\cosh^2 x - \sinh^2 x = 1$ (b) $(\cosh x)' = \sinh x$

5. Ecologists have discovered that the function $f(x) = x^m e^{-nx}$ where m and n are positive constants and $x \geq 0$ is important in the study of predator – prey relationships. Determine the values of x at which maxima and minima occur and the locations of points of inflection on the graph of f. Sketch the graph.

6. Student loans can take a very long time to pay off. If r is the annual interest rate on a loan and you make a monthly payment of \$$P$, then the Balance you owe at time t changes at a rate given by $B'(t) = \frac{r}{12} B(t) - P$. If the amount you borrow is \$$B_0$, then $B(0) = B_0$ assuming $t = 0$ corresponds to the time you graduate.

    (a) Find an explicit expression for $B(t)$; that is, solve the differential equation to obtain an two expression for $B(t)$ in terms of t.
    (b) How long will it take to pay off the loan?
    (c) If you borrow \$75,000 at an annual rate of 5% and make monthly payments of \$350, how old will you be when the loan is fully paid off? [You may use a calculator or *Maple* for this part].