MATH 122A Calculus II
Sample Examination 1

1. (a) Explain the difference between a Lorenz Function and a Gini Index.
(b) Find the Gini Index if

$$L(x) = \begin{cases} \frac{3}{4}x & \text{if } 0 \le x \le \frac{1}{3} \\ \frac{27}{16}\left(x - \frac{1}{3}\right)^2 + \frac{1}{4} & \text{if } \frac{1}{3} \le x \le 1 \end{cases}$$

(c) Suppose $L(x) = x^p$ for some constant p. Determine the value of p if the Gini Index = $\frac{1}{2}$
(d) Using your answer for (c), write an expression for the portion of the total wealth owned by the top 5% of the population.

2. Let f be the function defined by $f(x) = \frac{3}{8+x^3}$ on the closed interval I = [-1,4].
(a) Show that f is a one-to-one function.
(b) If g is the inverse of f, determine $g'(\frac{1}{3})$
(c) Find f' and determine the maximum and minimum values of f on the interval I.
(d) Determine where the graph of f is concave up and where it is concave down. Identify all points of inflection.
(e) Sketch a graph of f.

3. Differentiate each of the following functions with respect to x:
(a) $P(x) = \ln(\sin x)$ (b) $Q(x) = e^{x^2+7x}$ (c) $R(x) = log_8(\cos x)$
(d) $S(x) = \ln(e^x)$ (e) $T(x) = x^x$ (f) $U(x) = \int_2^x \frac{\sin t}{t}\, dt$
s(g) $V(x) = \int_1^{\sqrt{x}} \frac{3}{8+t^2}\, dt$

4. Suppose all we know about a continuous function f is that $f(0) = 0, f(1) = 1, f'(x) > 0$, all x, and $\int_0^1 f(x)\, dx = \frac{1}{4}$.
Find $\int_0^1 f^{-1}(x)\, dx$ where f^{-1} is the inverse of f.

5. (a) Give a careful statement of the Fundamental Theorem of Calculus.
(b) What is the definition of the natural logarithm function?
(c) How is the number e defined?

6. The amount x of light affects the rate y of photosynthesis by the relationship

$$y = f(x) = x^a\, e^{\left(\frac{a}{b}\right)(1-x^b)} = x^a exp\left(\left(\frac{a}{b}\right)1 - x^b\right)$$

where $x > 0$ and a and b are positive constants . Show that f has a maximum at $x = 1$.