Math 121 - Calculus I Name: solutions

Exam 2: Practice Exam

Please be sure to neatly **show and explain all of your work** and clearly label your answers. Except for your index card, this exam is a closed-book, closed-notebook exam. Calculators are not allowed.

Please write and sign the Honor Pledge here when you are done:

Signed:

Problem	Points .
1	/12
2	/10
3	/10
4	/8
5	/10
6	/10
Total	/60

1. Please compute the following. Show all work. You do not need to simplify your final answer.

(a) If $g(t) = \dfrac{\ln t}{t^2 + 3\tan t}$, what is $g'(t)$?

(quotient rule)

$$g'(t) = \frac{(t^2 + 3\tan t)\left(\frac{1}{t}\right) - (\ln t)(\cancel{t}2 + 3\sec^2 t)}{(t^2 + 3\tan t)^2}$$

(b) If $f(x) = e^x \cos(x^2)$, what is $\dfrac{df}{dx}$?

$$\frac{df}{dx} = \overbrace{-e^x \underbrace{\sin(x^2)(2x)}_{\text{chain rule}} + e^x \cos(x^2)}^{\text{product rule}}$$

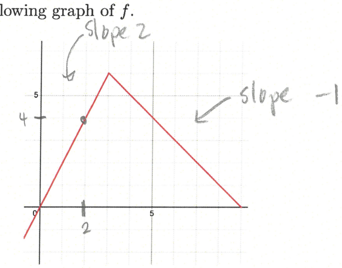
(c) $h(w) = \ln(w)\sin(w)$, what is $\dfrac{d^2h}{dw^2}$?

" $\ln(w)\cos(w) + w^{-1}\sin(w)$

$$\frac{dh}{dw} = \ln(w)\cos(w) + \frac{1}{w}\sin(w)$$ (product rule)

$$\boxed{\frac{d^2h}{dw^2} = -\ln(w)\sin(w) + \frac{1}{w}\cos w + \frac{1}{w}\cos(w) - \frac{1}{w^2}\sin w}$$

2. Consider the following graph of f.



Let

$$F(x) = f(f(x)).$$

Please use the graph of f to find the value of $\frac{dF}{dx}$ at $x = 2$. Show all work and explain your reasoning.

By the chain rule,

$$\frac{dF}{dx} = F'(x) = f'(f(x))f'(2).$$

From the graph, $f(2) = 4$.

So $F'(2) = f'(4)\,f'(2)$

$= (-1)(2)$

$= \boxed{-2.}$

3. Suppose that

$$f(x) = \frac{1}{x+3}.$$

(a) Please use **the limit definition of the derivative** to compute $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{x+h+3} - \frac{1}{x+3}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{x+3 - (x+h+3)}{(x+h+3)(x+3)}\right]$$

$$= \lim_{h\to 0} \frac{x+3 - x - h - 3}{h(x+h+3)(x+3)}$$

$$= \lim_{h\to 0} \frac{-h}{h(x+h+3)(x+3)}$$

(careful with)

$$= \lim_{h\to 0} \frac{-1}{(x+h+3)(x+3)}$$

$$= \frac{-1}{(x+3)(x+3)}$$

$$= \boxed{\frac{-1}{(x+3)^2}}$$

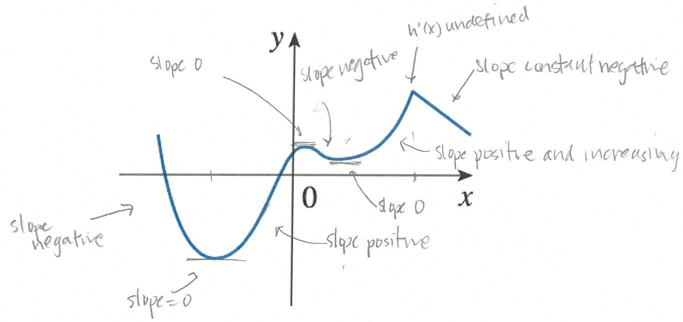
(b) Check your work: please find $f'(x)$ using a computational formula.

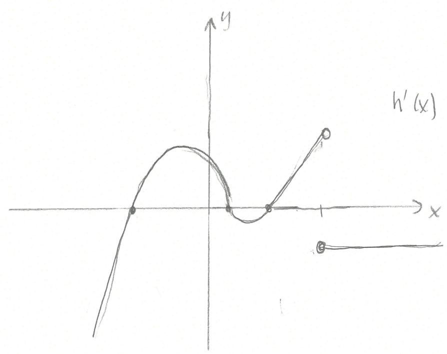
By the quotient rule,

$$f'(x) = \frac{(x+3)(0) - 1(1)}{(x+3)^2}$$

$$= \boxed{\frac{-1}{(x+3)^2}}$$ ✓

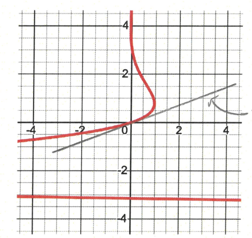
4. Consider the following graph of $h(x)$. Below the graph of $h(x)$, sketch a graph of the derivative of $h(x)$. Please indicate your reasoning.





5. Consider the curve defined by the equation

$$xe^y = 3\sin y.$$



slope $= \left.\frac{dy}{dx}\right|_{(0,0)} = \frac{1}{3}$.

(a) Explain (algebraically) how you know that the point $(0,0)$ lies on this curve.

If we plug $(0,0)$ into both sides of the equation:

$(0)e^0 = 0$ and $3\sin 0 = 0$,

we see that they are equal, so the $(0,0)$ satisfies the equation and thus lies on the curve.

(b) Please find the slope of the tangent line to the curve at $(0,0)$. Show all work.

By implicit differentiation.

$$\underbrace{\overbrace{xe^y\frac{dy}{dx}}^{\text{chain rule}} + e^y}_{\text{product rule}} = \overbrace{3\cos y\frac{dy}{dx}}^{\text{chain rule}}$$

Solving for $\frac{dy}{dx}$, we get

$$e^y = 3\cos y\frac{dy}{dx} - xe^y\frac{dy}{dx}$$

$$e^y = (3\cos y - xe^y)\frac{dy}{dx}$$

so $\frac{dy}{dx} = \frac{e^y}{3\cos y - xe^y}$.

slope at $(0,0)$

Thus $\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{e^0}{3\cos 0 - 0e^0}$

$$= \frac{1}{3(1) - 0(1)} = \frac{1}{3}$$

6. Find the equation of the tangent line to the graph of

$$k(z) = \cos((\ln z)^2)$$

at the point $z = 1$. Show all work.

point : $(1, k(1)) = (1, \cos((\ln 1)^2)) = (1, \cos(0^2)) = (1,1)$

slope : $k'(1)$.

chain rule with 3 layers.

$k'(z) = -\sin((\ln z)^2) \cdot 2(\ln z) \cdot \frac{1}{z}$

$k'(1) = -\sin((\ln 1)^2) \cdot 2(\ln 1) \cdot \frac{1}{1}$

$= -\sin(0^2) \cdot 2(0) \cdot 1$

$= 0 \cdot 0 \cdot 1$

$= 0$. ← slope 0.

Equation of line: $y = mx + b$

$1 = 0(1) + b \Rightarrow b = 1$

Line: $\boxed{y = 1}$ $(y = 0x + 1)$