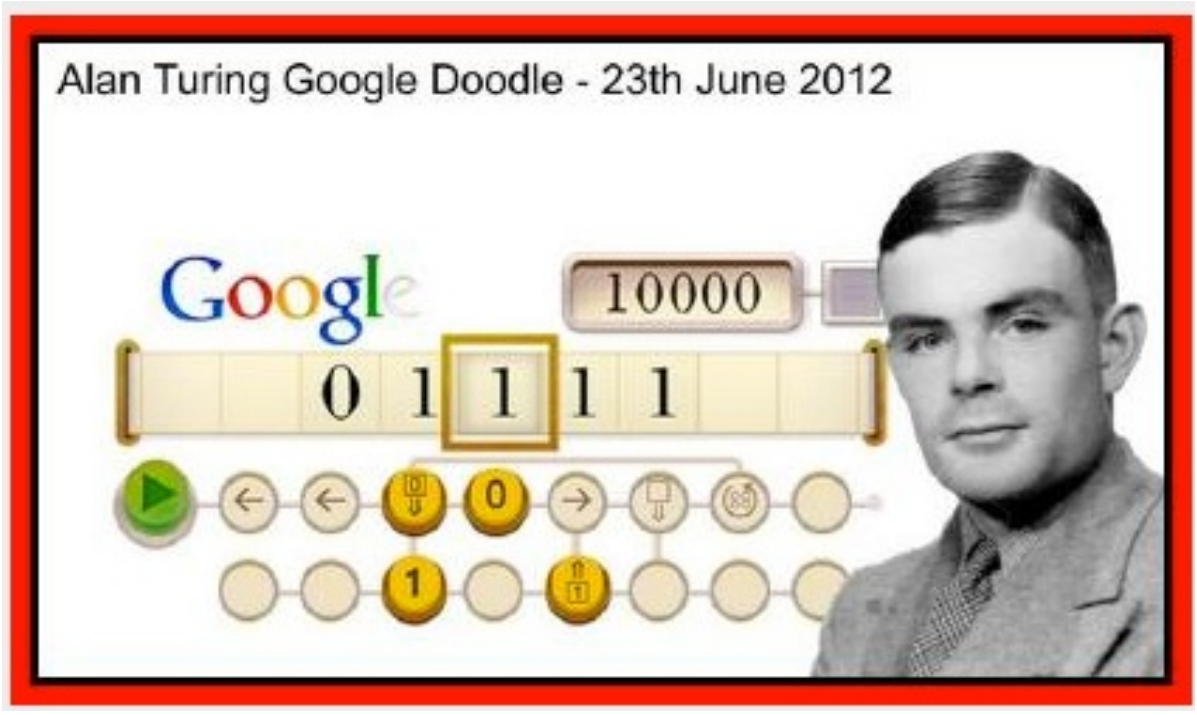
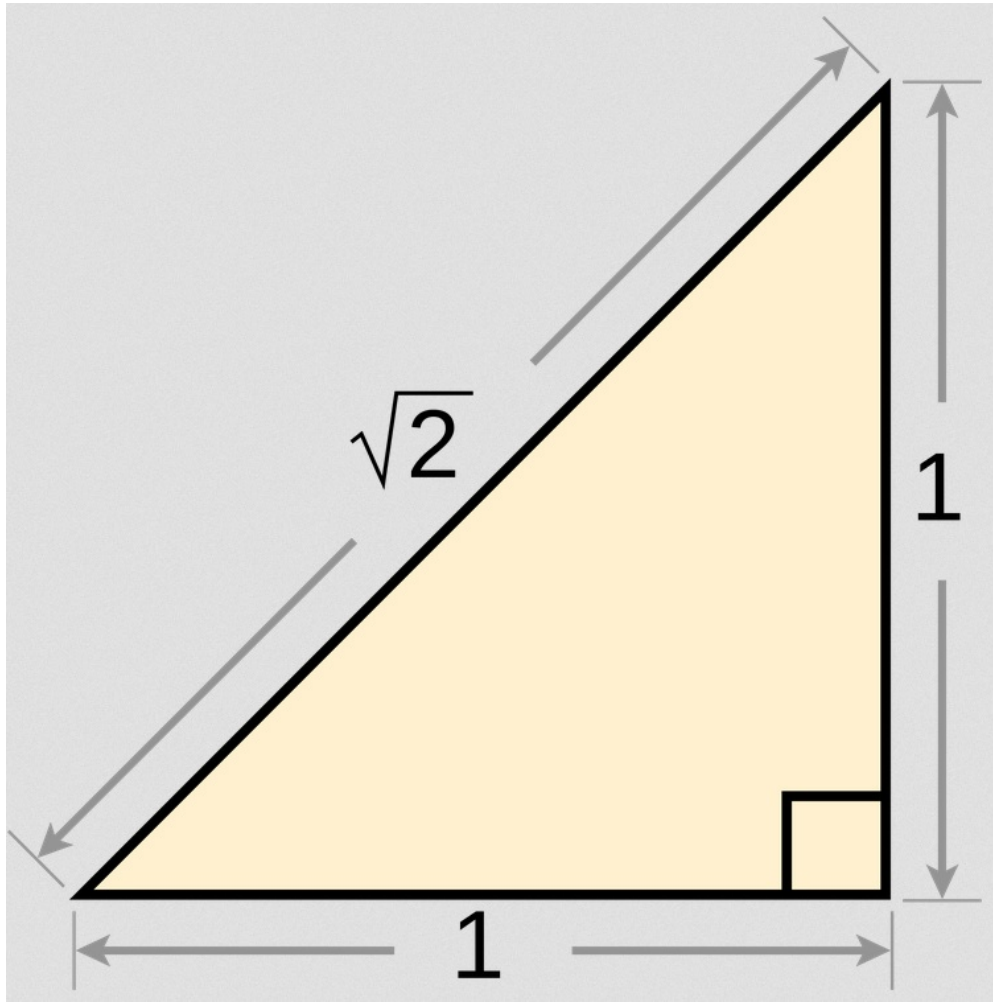

# Some Notes on Assignment 13
# Assignment 14
# Some Notes on Exam 1

# Turing and Numbers II



Is there a **naturally occurring** number that is not rational?

The square root of 2 is not rational.
(Hypotenuse of a right triangle of sides 1)



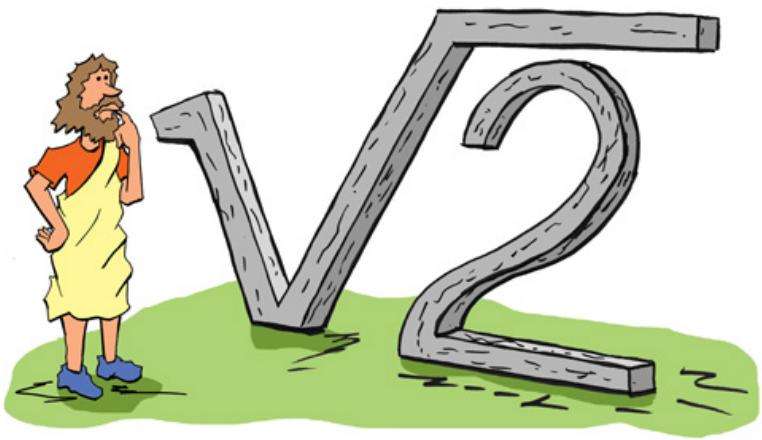
# The First Great Crisis in Mathematics

# Pythagoras of Samos

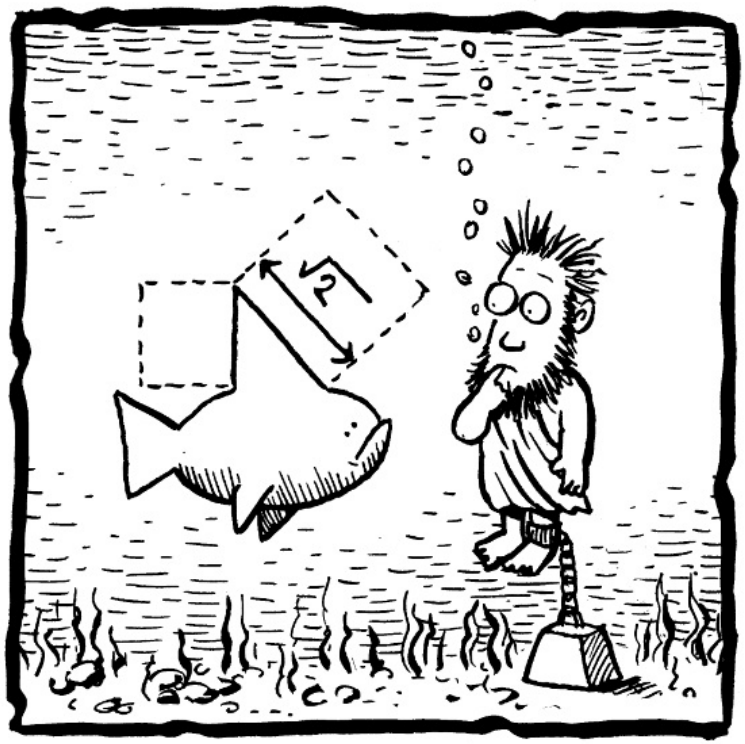
Link to More About Pythagoras and Pythagoreans

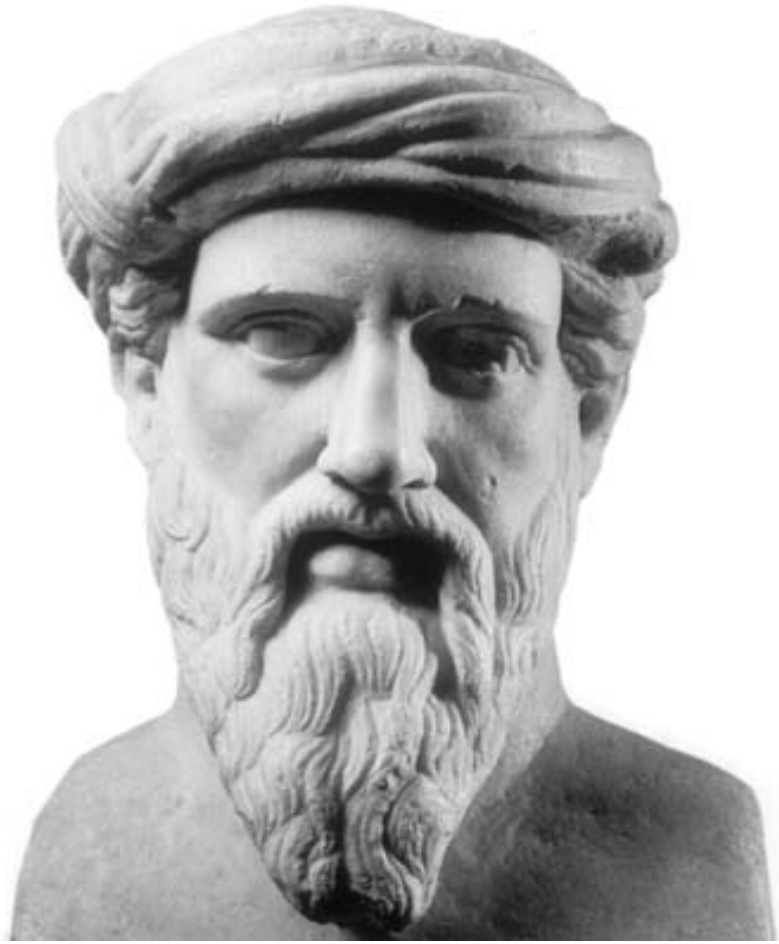
The first person to recognize the existence of irrational numbers might have died for his discovery. **Hippassus of Metapontum** was an Ancient Greek philosopher of the Pythagorean school of thought. Supposedly, he tried to use his teacher's famous theorem $a^2 + b^2 = c^2$ to find the length of the diagonal of a unit square. This revealed that a square's sides are incommensurable with its diagonal, and that this length cannot be expressed as the ratio of two integers. The other Pythagoreans believed dogmatically that only positive rational numbers could exist. They were so horrified by the idea of incommensurability, that they threw Hippassus overboard on a sea voyage, and vowed to keep the existence of irrational numbers an official secret of their sect.

Before



After





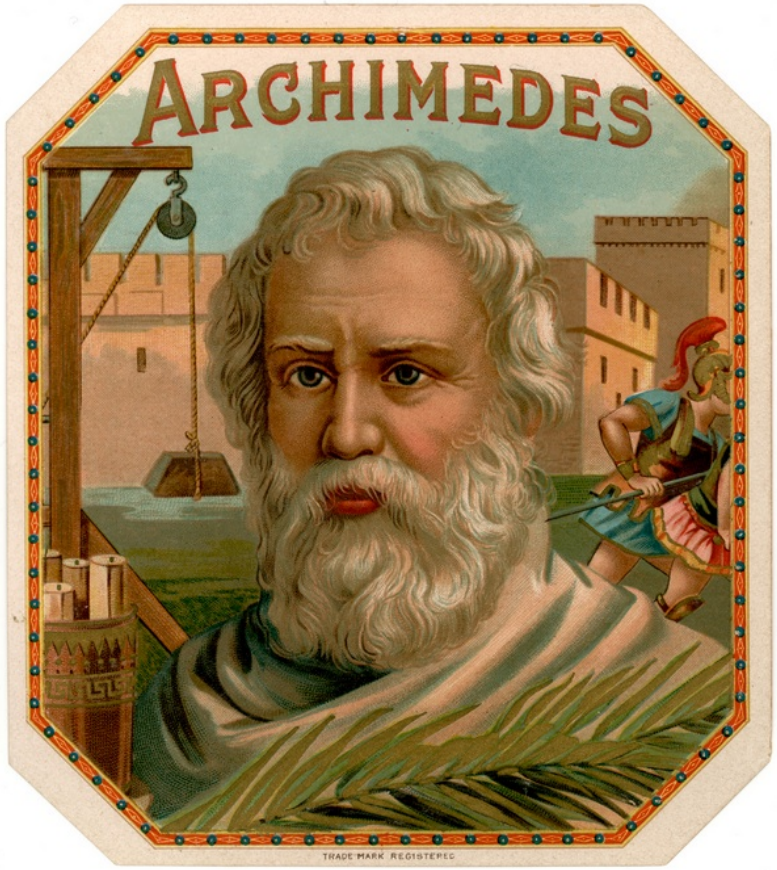
*The School of Athens* by Raphael

Born: about 569 BC in Samos, Ionia

Died: about 475 BC

Link to Biography of Pythagoras

# Archimedes of Syracuse




Born: 287 BC in Syracuse, Sicily

Died: 212 BC in Syracuse, Sicily

Link to Archimedes Biography

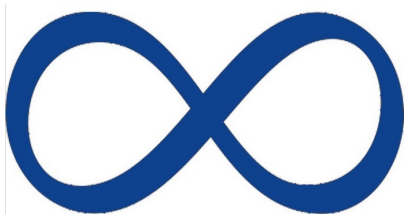
## Greek Abhorrence of Infinity?

*Potential vs Actual Infinite (Aristotle)*

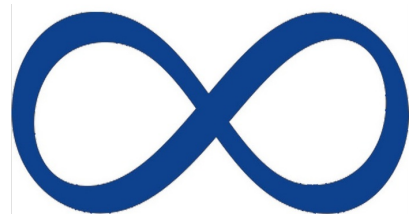
Aristotle distinguished between actual and potential infinities. An actual infinity is something which is completed and definite and consists of infinitely many elements. A potential infinite is a sequence which is endless. Whereas all the elements of an actually infinite set are assumed to exist together simultaneously, the elements of a potentially infinite sequence exist only consecutively over time.

Archimedes Palimpset

For more details, link here Palimpset



# INFINITY

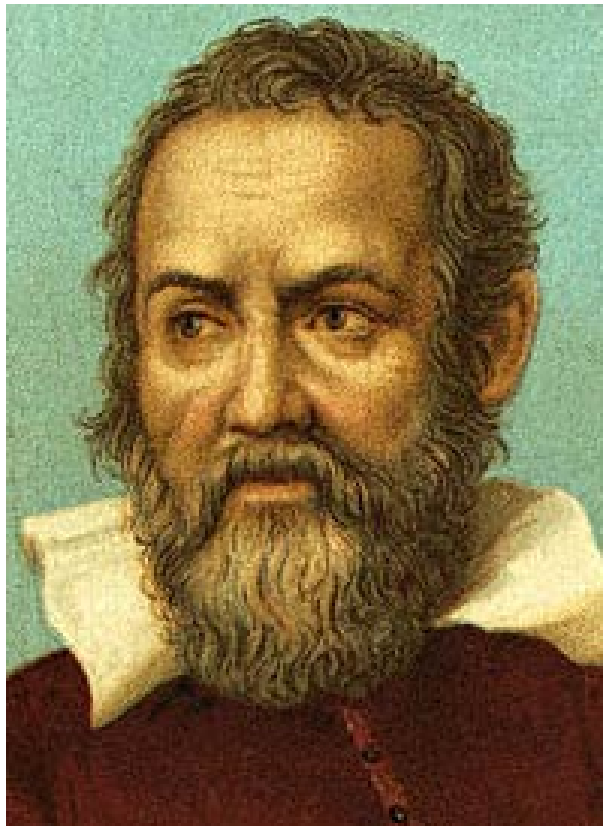


# GALILEO'S PARADOX

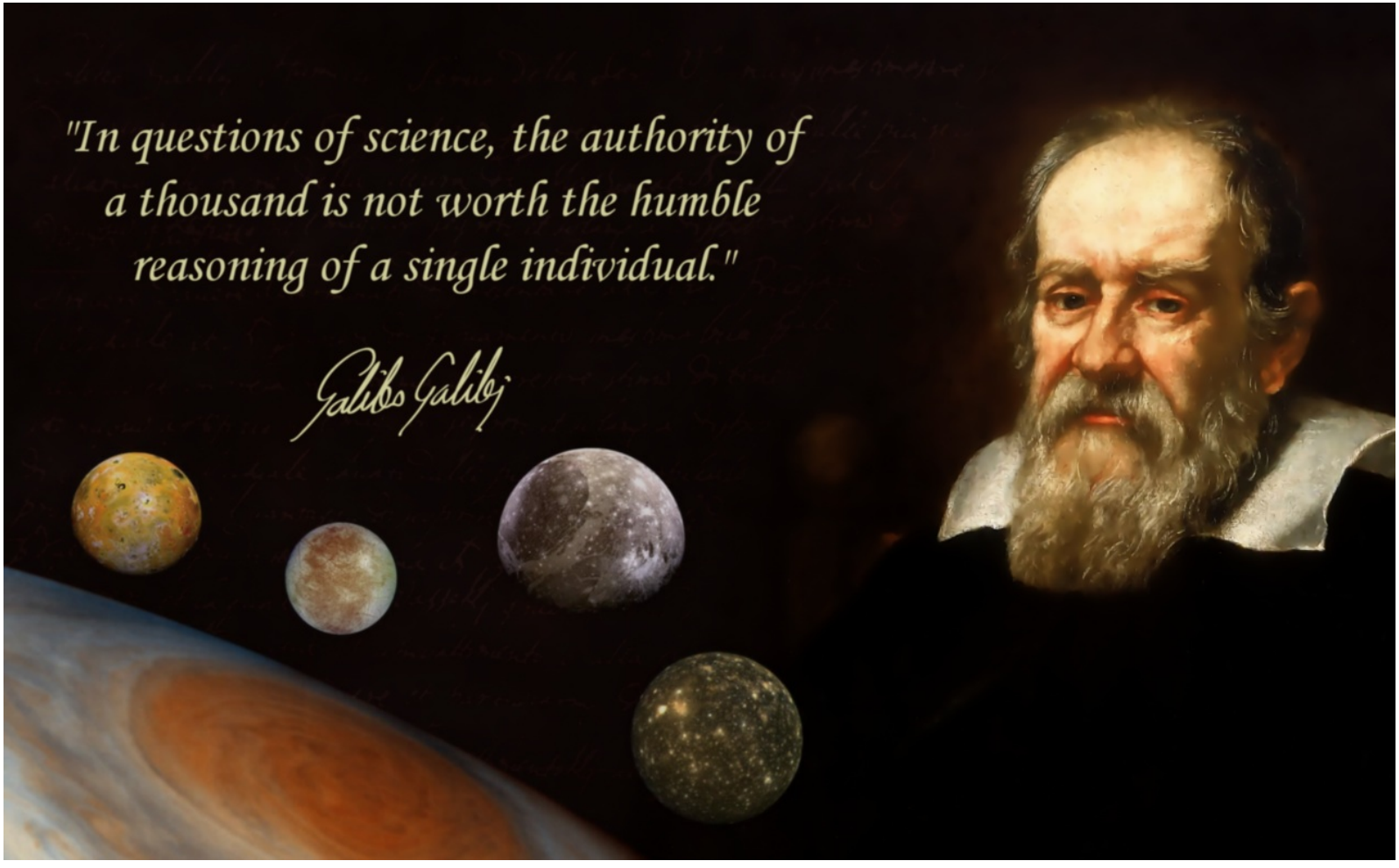
The whole is greater than any of its parts?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | . . . |
|---|---|---|---|---|---|---|---|---|---|
| ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | ↕ | |
| 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | |

# Galileo Galilei



Born: February 15,1564 in Pisa
Died: January 8, 1642 in Arcetri (near Florence)
Galileo Biography Link


"In questions of science, the authority of
a thousand is not worth the humble
reasoning of a single individual."
Galileo Galilei

*Galileo Facing the Roman Inquisition* by Cristiano Banti (1857).

# Bernard Placidus Johann Nepomuk Bolzano

Born: October 5,1781 in Prague

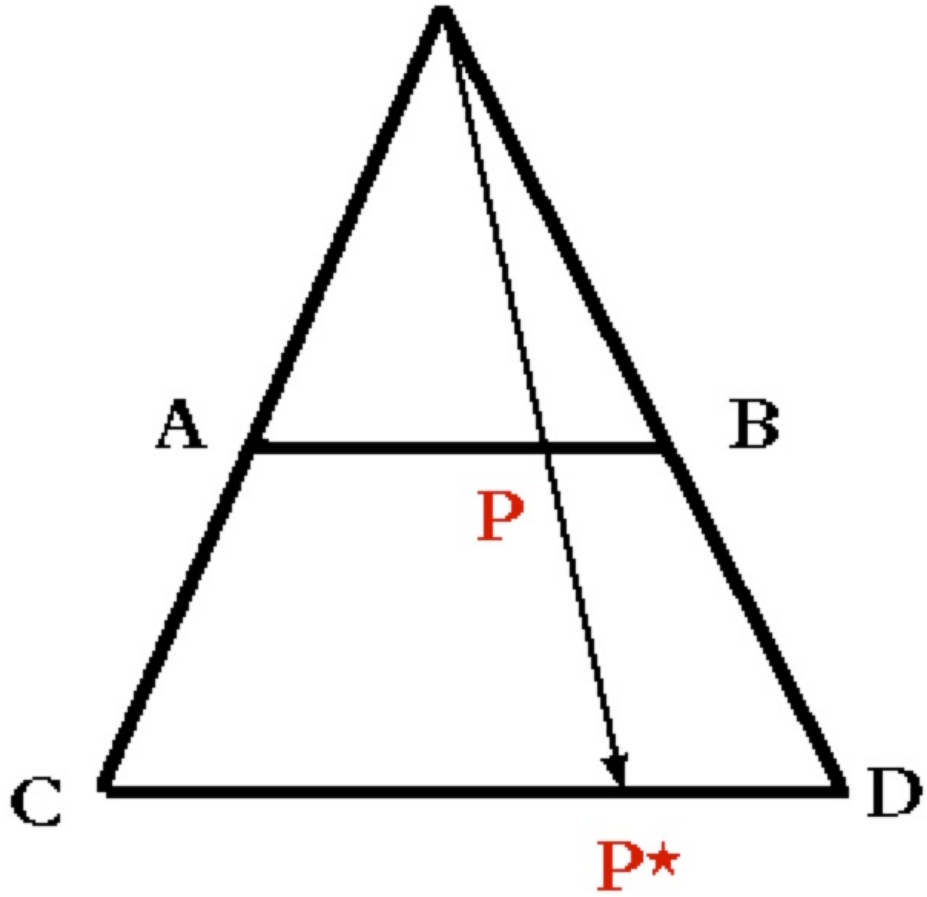
Died: December 18, 1848 in Prague

Bolzano Biography Link

*Paradoxien des Unendlichen*(1851)

*Paradoxes of the Infinite*

# Bolzano's Paradox




**The line segment CD is twice as long as AB but has the same number of points!**

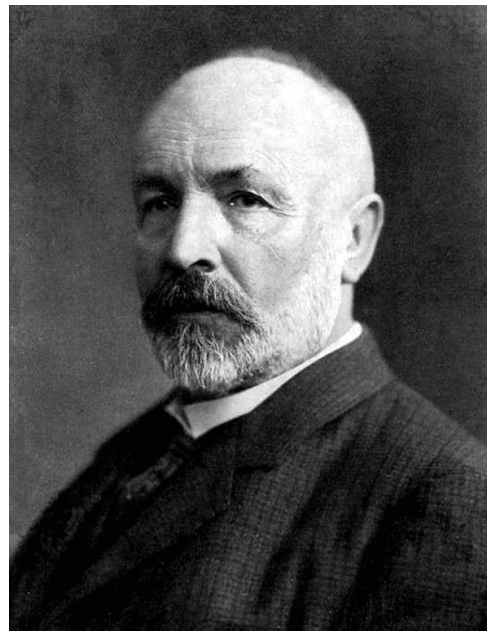
# MID 19TH CENTURY VIEW OF INFINITY

A collection is **finite** if it has $n$ elements for some non-negative integer $n$.

A collection is **infinite** if it is not finite.

Cantor Turned This Upside Down

# Georg Ferdinand Ludwig Philipp Cantor



Born: March 3, 1845 in St Petersburg, Russia
Died: January 6, 1918 in Halle, Germany
Cantor Biography Link

**According to Cantor:**

A collection is **infinite** if it can be put into a one-to-one correspondence with one of its subsets.

A collection is **finite** if it is not infinite.

Two collections **have the same size** if there is a one-to-one correspondence between the elements of each collection.

## Examples

Counting Numbers = Positive Integers

Even Positive Integers = 2,4,6,8,10,...

Odd Positive Integers = 1,3,5,7,9,11,?

Primes = 2, 3, 5, 7, 11, 13, 17, 19, 21, 23, 29,?

Integers: 0, +1, -1, +2 , -2, +3, -3, . . .

Rational Numbers

Real Numbers