

Exam Tonight





7 PM in Warner 010

No Books, Notes, Computers, Cell Phones, etc.
But All The Time You Need





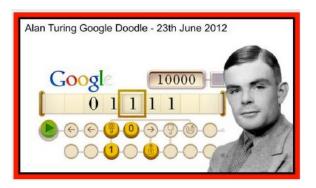
Don't have time for lunch?

Spending 30 minutes in line for food you have to gobble up in a stressful hurry on the way to your next class?

"I have a couple solutions to this. Ross is open until 2:30, so hopefully students can head up there at 2 and eat. Also, we do have a to-go container program and set up the deli bar first thing in the morning in case students need to make a sandwich at breakfast and bring it with them if they do not have time to get to the dining halls [midday]."

Dan Deltora, Head of Dining Services.

Turing and Numbers



Numbers

How to represent numbers?

Individual slashes (unary system) ||||||||

Roman Numerals: Decimal Notation Ten symbols: What does a

notation like A = 76451 mean?

$$A = 7 \times 10^4 + 6 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 1 \times 10^0$$

Beyond the decimal point:

How about 76451.238 ?

$$A + 2 \times 10^{-1} + 3 \times 10^{-2} + 8 \times 10^{-3}$$

Hexadecimal (base 16): 16 symbols:



Binary: two symbols 0,1 Translating from Binary to Decimal Translating from Decimal to Binary

Types of Numbers:

Natural Numbers

Non-negative Integers

Integers

Rational Numbers: quotients of integers.

Real Numbers: all decimals or points on a line

Rational Numbers: Repeating Decimals

A number is rational if and only if its decimal representation eventually repeats

Find decimal representation of 3/7

What rational number has decimal representation X = 12.985985985985...?

$$1000X = 12985.985985985985...$$

 $X = 12.985985985985...$
 $999X = 12973$
 $X = \frac{12973}{999}$

$$1/3 = .33333333333...$$

 $1 = .9999999...$
 $1.0000... = .9999999...$

A number may have two different decimal representations.

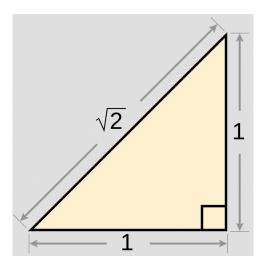
Are there some numbers which are not rational?

To write down one, just create a decimal representation with no block that keeps repeating:

Example: 0.1001000100001000001000001...

Is there a **naturally occurring** number that is not rational?

The square root of 2 is not rational. (Hypotenuse of a right triangle of sides 1)





G. H. Hardy

(From Andrew Hodges biography of Turing)
Alan responded with joy to the absolute quality of mathematics, its apparent independence of human affairs, which G. H. Hardy expressed another way:

317 is a prime, not because we think so, or because our minds are shaped in one way rather than another, but because it is so, because mathematical reality is built that way.

(G. H. Hardy, A Mathematician's Apology, 1940)

The Square Root of 2 is Irrational.

Proof: Suppose Not (a proof by contradiction).

Suppose there are integers a and b such that $\frac{a}{b} = \sqrt{2}$.

We may assume that the fraction is in lowest terms; in particular, not both a and b are even. At least one of them must be odd.

Now $(\frac{a}{b})^2 = 2$ or $\frac{a^2}{b^2} = 2$ so $a^2 = 2b^2$.

But $2\vec{b}^2$ is even so a^2 must be even.

If a were odd, its square a^2 would also be odd; Hence a is even.

There is an integer m so that a = 2m which implies that $a^2 = 4m^2$.

Our equation $a^2 = 2b^2$ can be written as $4m^2 = 2b^2$ so $2m^2 = b^2$ which makes b^2 even.

Hence b must also be even.

We have a contradiction since at least one of a or b had to be odd.

Godfrey Harold Hardy



Born: February 7, 1877 in Cranleigh, Surrey, England Died: December 1, 1947 in Cambridge, Cambridgeshire, England Hardy Biography Link

The First Great Crisis in Mathematics