

**Code Makers and Code Breakers
From Julius Caesar To Alan Turing
And Beyond V**

**An Introduction To
Some Mathematics and Linguistics
Used to Crack Secret Ciphers**

ADA LOVELACE

CHARLES BABBAGE

THE VIGENÈRE CYPHER



Detecting a Polyalphabetic Cipher

A monoalphabetic cipher has greater variation among frequencies of the individual letters



William Friedman (1930)
Index of Coincidence

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$$\sum_{i=A}^Z \frac{f_i(f_i - 1)}{N(N - 1)}$$

f_i = frequency of letter i

N = number of characters in message

Number of Alphabets	Expected IC
1	.066
2	.052
5	.044
10	.041
26	.038

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If the keyword has n letters and some word appears in the plaintext more than n times, then that word will be enciphered the same way at least twice in the cipher text.

How Can We Make a Vigenère Cipher More Secure?

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Try To Get Rid of Repetitions

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**Take Keywords Longer
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Examples

Declaration Of Independence
The Bible

Try This At Home:

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3. Use our Vigenère program to generate the ciphertext
4. How flat is the frequency count for the cipher text?
5. What is the longest string in the cipher text that repeats itself?

Possible Weaknesses of This Approach

IS THERE A THEORETICALLY UNBREAKABLE CIPHER SCHEME?

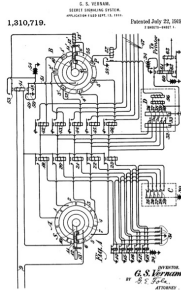
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THE ONE TIME PAD

Each character in plaintext is shifted by a random number



Gilbert Vernam
1890 – 1960



Vernam's Patent Figure

USING A ONE TIME PAD

11	25	20	14	13	12	9	18	21	5	24	17	18	9	5
12	8	2	1	12	9	14	13	15	23	17	9	21	14	17
20	15	24	5	17	21	10	21	5	5	26	3	22	22	23
1	4	14	18	24	2	18	4	17	18	21	22	10	5	21
1	13	2	8	26	2	5	1	3	25	2	2	12	19	21
4	4	21	8	12	11	7	17	18	14	20	18	18	19	12
2	22	20	14	10	20	4	13	23	13	5	1	5	6	14
14	16	3	8	16	9	9	14	14	21	3	17	25	24	15
26	23	12	19	23	3	8	4	17	23	17	6	24	20	16
17	4	10	15	16	26	14	3	2	22	10	1	22	23	17
13	18	25	24	10	17	14	12	26	23	15	7	5	26	19
21	21	3	16	16	20	9	21	12	23	16	21	1	6	10

M	A	T	H	E	M	A	T	I	C	S
13	1	20	8	5	13	1	20	9	3	19
11	25	20	14	13	12	9	18	21	5	24
24	26	40	22	18	25	10	38	30	8	43
24	26	14	22	18	25	10	12	4	8	17
X	Z	N	V	R	Y	J	L	D	H	Q

Vernam One Time Pad in Binary

Add Modulo 2 with no carries; Also called XORing

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 0$$

Letter	Position	Binary
A	1	00001
B	2	00010
C	3	00011
D	4	00100
E	5	00101
F	6	00110
G	7	00111
H	8	01000
I	9	01001
J	10	01010
K	11	01011
L	12	01100
M	13	01101
N	14	01110
O	15	01111
P	16	10000

Letter	Position	Binary
Q	17	10001
R	18	10010
S	19	10011
T	20	10100
U	21	10101
V	22	10110
W	23	10111
X	24	11000
Y	25	11001
Z	26	11010
[SPACE]	27	11011
.	28	11100
,	29	11101
!	30	11110
:	31	11111
;	32	00000

plaintext: **T****H****I****S** IS THE MESSAGE

One Time Pad: 01010100100100011110001111000011?

0	1	0	1	0	1	0	0	1	0	0	1	0	0	0
1	0	1	0	0	0	1	0	0	0	0	1	0	0	1
1	1	1	1	0	1	1	0	1	0	0	0	0	0	1

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0	1	0	1	0	1	0	0	1	0	0	1	0	0	0
1	0	1	0	0	0	1	0	0	0	0	1	0	0	1
1	1	1	1	0	1	1	0	1	0	0	0	0	0	1

11110 becomes !
 11010 becomes Z
 00001 becomes A
 Ciphertext begins **!ZA**



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- ▶ You must use the pad only once.
- ▶ Creation, Distribution, and Storage of pads are a problem.
- ▶ How many pads would the State Department require?

THE



MACHINE





Think of the Enigma Machine as a Vigenère
Cipher With a Keyword of Length

$$26 \times 26 \times 26 = 17,576$$