

Assignment 16
For Friday, October 24



Hut 8



Gordon Welchman

Reading

Read Chapter 5 “History of Hut 8 to December 1941 featuring an excerpt from Turing’s ‘Treatise on the Enigma’” by Patrick Mahon And Chapter 7 “Letter to Winston Churchill” in *The Essential Turing*.

Writing

Finish work on Essay 2 which is due on Monday, October 27.

Problems

- I. Consider a rule for pairing the set $\mathbf{R}^+$ of non-negative real numbers with the set $\mathbf{I}$ of real numbers between 0 and 1: If x is a number between 0 and 1, pair it with $x/2$ but if x is larger than 1, pair it with $1 - \frac{1}{2x}$. Mathematicians would write this correspondence as a function f defined by the rule $f(x) = \begin{cases} \frac{x}{2} & \text{for } 0 \leq x \leq 1 \\ 1 - \frac{1}{2x} & \text{for } x > 1 \end{cases}$
- Show that this rule gives a one-to-one correspondence between $\mathbf{R}^+$ and $\mathbf{I}$ so that these two sets have the same size.
- II. Let L be the set of points on the real line and P the set of points in the plane. Using Cantor’s definitions,
- 1) Show that L is an infinite set.
 - 2) Show that P is an infinite set.
 - 3) Find a one-to-one correspondence between L and a proper subset of P .